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# Effect of Uniform and Space-Dependent Heat Source on the Onset of Buoyancy-Driven Convection in Viscosity Fuels: A Linear Theory

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## Abstract

*The viscosity of fuel oil is significantly influenced by temperature, with higher temperatures leading to lower viscosity. To ensure optimal combustion, it's crucial to maintain the fuel's viscosity within a specific range. With regard to variable, space-dependent and uniform heat sources, the impact of variable viscosity on the stability of Buoyancy Rayleigh-Bénard convection is demonstrated. The impact of non-inertial acceleration on natural convection is also studied in the problem. The Fourier series representation of stream function, temperature distribution describes how to derive an analytical expression for the thermal Rayleigh number. Here we noticed that the heat source parameter, the viscosity parameter, and the Taylor number effect the stability of the fluid system. Also, it is demonstrated here the impact of rotational strength accompanied with the stabilized system, where as an increase in the internal Rayleigh number and thermorheological parameter is to destabilize the same. It is also observed that, it is possible to control convection by proper tuning these parameters. A comparative study of external Rayleigh number and stability analysis for the onset of instability is presented in the problem. Some of the important new results have been revealed in the context of heat sources*

**Keywords:** *Heat Source/Sink, Rayleigh-Benard Convection, Taylor Number, Variable Viscosity Fuels*

## 1.0 Introduction

Fuel viscosity plays a pivotal role in shaping various aspects of fuel injection, spray formation, and combustion in Compression Ignition (CI) engines. In contemporary diesel engine injector designs, such as rail injection systems, which operate at extremely high pressures exceeding 100 MPa, the viscosity of fuel surpasses atmospheric values by a significant margin. Precisely estimating the viscosity of biodiesel (BD) based on its composition shows great potential for optimizing biodiesel production processes, particularly when blending different raw materials and refined products. Natural convection issues associated with buoyancy, heat source/sink, thermorheological effects, and Coriolis forces have been explored by numerous researchers. Over recent decades, the quest for stabilizing systems or controlling heat transfer has been a compelling challenge due to their vast applications in geophysics and astrophysics. The classic Benard convection driven by buoyancy has been extensively studied and well-documented in various authoritative works, including those authored by Chandrasekhar[1], Nakagawa[2], Platten and Legros[3], and Drazin and Reid[4]. The external mechanisms that suppress the onset of convection is rotation, the strength of rotational force is represented by a non-dimensional Taylor number. Vadasz[5] consider the same problem and made a stability analysis using linear theory under the

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influence of Coriolis effect in a porous layer and found that rotation is to have a stabilizing effect. Besides this examination, the study made by Ecke et al.[6] also concludes the same. Kiran and Bhadauria[7] made a weakly non-linear stability analysis of Buoyancy convection under the influence rotation, this study explains how Taylor number can be tuned in order to suppress the onset of convection and heat transfer (Kapil et al.[8] and Venugopal and Arnab Kumar[9]). Recently, King et al.[10] demonstrated some important results on scaling behavior of Rayleigh-Bénard convection with rotation. The material offers its heat source in many engineering and science problems, resulting in enhanced heat transfer. Such a situation can even occur in various fields, such as radioactive decay, chemical reactions, and nuclear reactions. It can even occur in some celestial bodies that are kept warm and active by nuclear reactions and radioactivity. Because of heat source/sink, there would be a temperature gradient between the center of Earth and surface that helps maintain convective heat flux, thus transferring heat energy toward the Earth's surface. Therefore, the internal heat source/sink role is very important for various applications such as radioactive materials, geophysics, astrophysics, etc. However, some relevant studies have investigated the effects of internal heat source on convective flow in a fluidized layer. McKenzie et al.[12] studied natural convection due to Buoyancy in the Earth's mantle taking large value of Prandtl number explains geophysical information and convection in a Boussinesq fluid. (Tveitereid and Palm[13], Clever[14], Riahi et al.[15,16], Siddheshwar and Titus[17] and Srivastava et al.[18]).

Viscosity is one important fluid property represents ratio between shear stress and strain. Viscosity is temperature dependent in many situations, even at room temperature. It is one of the fluid properties that indicate the convection phenomenon. The impact of temperature-structured variable viscosity in an exponential model using truncated Taylor series turned into studied via Torrance and Turcotte[19]. Straughan[20] demonstrated the consequences of temperature-dependent viscosity on convection and located that thermorheological parameters affect the onset of convection and reason destabilization of the system. Siddheshwar et al.[21], Wu and Libchaber[22] and Shateyi and Motsa[23] have proven that the electrically conductive fluid in the presence of suspended solids is greater strong than the conventional electrically conductive fluid without suspended solids while the underlying fluid layer is heated, where the fundamental wave quantity is insensitive to the suspension parameters however more sensitive to the Chandrasekhar number. It is widely known that the viscosity of the Earth's mantle is strongly temperature structured (Giannandrea and Christensen[24], Booker[25]), which absolutely affects the convection sample. consequently, it is critical to do not forget this rheology in a laboratory or numerical studies.

Recently, the minimal representation of Fourier series expansion finds useful application in convection problems (Ramachandramurthy and Aruna[27,28]). Accordingly, this problem deals with the linear stability analysis of natural convection (Buoyancy-driven convection) in the temperature-dependent variable viscosity Newtonian liquid with uniform heat sources. We select the Fourier cosine series method to model the basic viscosity and temperature distribution. This approach is implemented to find the analytical expression for the thermal Rayleigh range as a function of internal Rayleigh number, thermorheological parameter, and Taylor number. The Galerkin method is utilized to analyze the boundary value problem. We illustrate comparisons of integrity and range of occurrence of thermal convection at the onset of the phenomenon as a function of space-dependent heat source data.

## 2.0 Mathematical Formulation for Space-dependent Heat Source

Consider a temperature-sensitive Newtonian liquid between two infinite parallel horizontally extended planes of known depth $d$ between $y=0$ and $y=d$ and finite temperature gradients between $y=0$ and $y=d$. The bottom and top sides are kept at fixed temperatures, and the bottom wall is rigid and stress-free with the condition that the surface $y=0$ is provided with a sloped in temperature. Considering only small convective movements (Lorentzian), we assume that the Oberbeck-Boussinesq approximation follows in which the boundary is stress-free and isothermal. The dynamic viscosity and density varies along the $y$-axis, and its heat source is assumed to vary along the $y$-axis. The governing equations for the present problems with the Coriolis force are given by

$$\nabla.\vec{q} = 0, \tag{1}$$

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}+2(\Omega\times\vec{q})\right]=-\nabla p+\rho(T)\vec{g}+\nabla.\left[\mu_f(T)\left(\nabla\vec{q}+\nabla\vec{q}^{Tr}\right)\right], \tag{2}$$

$$\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\chi\nabla^2T+Qv, \tag{3}$$

$$\rho(T)=\rho_0[1-\beta(T-T_0)], \tag{4}$$

$$\mu_f(T)=\mu_0 e^{-\delta(T-T_0)}. \tag{5}$$

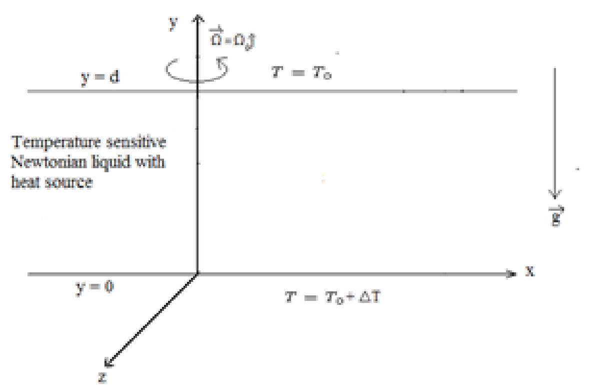


**Figure 1.** Schematic of Flow configuration.

In order to perform the linear stability analysis, we consider the following perturbations:

$$\vec{q}=q_b(y)+q',\ p=p_b(y)+p',T=T_b(y)+T' \\ \rho=\rho_b(y)+\rho',\mu=\mu_b(y)+\mu' \tag{6}$$

where the primes represents perturbation quantities. Prior to perturbation the fluid state is at rest and it is in conduction state, the basic state quantities $T_b(y)$, $\mu_{f_b}(y)$ and $\rho_b(y)$ have their solutions in the form

$$\vec{q}=0,T_b=T_0+\Delta Tf\left(\frac{y}{d}\right),\rho_b\left(\frac{y}{d}\right)=\rho_0\left[1-\beta\Delta Tf\left(\frac{y}{d}\right)\right],$$

$$p_b\left(\frac{y}{d}\right)=-\int p_b\left(\frac{y}{d}\right)gd\left(\frac{y}{d}\right)+C_0 \tag{7}$$

Where, $f\left(\frac{y}{d}\right)=1-\frac{y}{d}$

is linear uniform basic temperature gradient. One can obtain this expression by solving energy equation in the absence of time and the velocity components $u=v=w=0$. We restrict our study to two-dimensional flow, therefore it is necessary to introduce the stream functions the flow as follows:

$$u'=\frac{\partial\psi'}{\partial y},\quad v'=-\frac{\partial\psi'}{\partial x} \tag{8}$$

Eliminating the pressure from these Eqn. (2.2) using the classical procedure of taking curl twice, the momentum and energy equations takes the following form:

$$\rho_0\frac{\partial}{\partial\tau}(\nabla^2\psi')=\frac{\partial\mu_{f_b}}{\partial y}\frac{\partial}{\partial y}(\nabla^2\psi')+,$$

$$\mu_{f_b}\nabla^4\psi'-\rho_0\alpha g\frac{\partial T'}{\partial x}-2\rho_0\Omega_0\frac{\partial w'}{\partial y} \tag{9}$$

$$\frac{\partial T'}{\partial t}=-\frac{\partial\psi'}{\partial x}\frac{dT_b}{dy}+\chi\nabla^2T'+Q\frac{\partial\psi'}{\partial x}, \tag{10}$$

$$\rho_0\frac{\partial w'}{\partial t}=\mu_{f_b}{}'\nabla^2w'+2\rho_0\Omega_0\frac{\partial w'}{\partial y}+\frac{\partial(\psi',w')}{\partial(x,y)}. \tag{11}$$

Now non-dimensionalize the Eqs. (9)-(11) using

$$(X,Y)=\left(\frac{x}{d},\frac{y}{d}\right),\ \tau=\frac{\chi}{d^2}t,\ \Psi=\frac{\psi'}{\chi},\ \Theta=\frac{T'}{\Delta T}, \tag{12}$$

and we get the following non-dimensional momentum, energy and vorticity equations

$$\frac{1}{\Pr}\frac{\partial}{\partial\tau}(\nabla^2\Psi)=\frac{\partial\mu_{f_b}}{\partial Y}\frac{\partial}{\partial Y}(\nabla^2\Psi)+\mu_{f_b}\nabla^4\Psi+$$

$$R_E\frac{\partial\Theta}{\partial X}-\sqrt{Ta}\frac{\partial\zeta}{\partial Y}+\frac{\partial\left(\Psi,\nabla^2\Psi\right)}{\partial(X,Y)}, \tag{13}$$

$$\frac{\partial\Theta}{\partial\tau}=-\frac{\partial\Psi}{\partial X}+\nabla^2\Theta-R_I\frac{\partial\Psi}{\partial X}+\frac{\partial(\Psi,\Theta)}{\partial(X,Y)}, \tag{14}$$

$$\frac{1}{\Pr}\frac{\partial\zeta}{\partial\tau}=\mu_{f_b}\nabla^2\zeta+\sqrt{Ta}\frac{\partial\zeta}{\partial Y}+\frac{\partial(\Psi,\zeta)}{\partial(X,Y)}. \tag{15}$$

where $Ta = \left(\frac{2\Omega d^2}{\nu}\right)^2$ is the Taylor number due to rotation of the fluid, $\Pr = \frac{\gamma}{\chi}$ is the Prandtl number, $R_I = \frac{Qd}{\Delta T}$ is the internal Rayleigh number, $R_E = \frac{\alpha g \Delta T d^3}{\gamma \chi}$ is the external thermal Rayleigh number which is the eigenvalue of the problem. The boundary conditions at $Y = 0,1$ for solving the Eqs.(2.13), (2.14) and (2.15) are displayed below

$$\Psi = \nabla^2 \Psi = \Theta = D\zeta = 0 \tag{16}$$

## 3.0 Linear Stability Analysis

To perform linear theory, we consider the linearized version of Eqs. (13), (14) and (15) together with the boundary conditions (2.16). The solutions are assumed to be a periodic wave of the following form: (As discussed in Aruna *et al.*[30-32]).

$$\left.\begin{aligned} \Psi(X,Y) &= \Psi_0 \sin(\pi\alpha X)\sin(\pi Y) \\ \zeta(X,Y) &= \zeta_0 \sin(\pi\alpha X)\cos(\pi Y) \\ \Theta(X,Y) &= \Theta_0 \cos(\pi\alpha X)\sin(\pi Y) \end{aligned}\right\}. \tag{17}$$

Substitute these Fourier modes into Eqs. (13), (14) and (15) (linearized version) and using standard orthogonalization procedure, a set of a homogeneous linear system of equations of three unknowns is obtained. In the event of obtaining a non-trivial solution of the system, we get the following analytical expression for the eigenvalue $R_E$:

$$R_E = \frac{\eta_1^4 \left(\frac{a_0}{\pi} - \frac{2a_2}{3\pi} - \frac{2a_4}{15\pi}\right)\left(\frac{\eta_1^2}{2} a_0 + (1 - \frac{\eta_1^2}{2}) a_2\right) + 2\pi Ta}{\pi^2 \alpha^2 \left(\frac{a_0}{\pi} - \frac{2a_2}{3\pi} - \frac{2a_4}{15\pi}\right)(R_I + 1)} \tag{18}$$

where $\eta_1^2 = \pi^2(1+\alpha^2)$, $a_0 = 2\mu_0 \int_0^1 e^{V(Y-1)} dY$,

$a_n = 2\mu_0 \int_0^1 e^{V(Y-1)} \cos n\pi Y dY$.

To derive the above expression we have applied standard orthogonal procedure. The scaled wave number $\alpha_c$ satisfies

$$(a_0 - a_2)\pi^4\alpha^6 + \frac{1}{2}\left(3a_0\pi^4 - a_2\pi^4 - R_I a_0 \pi^2 + R_I a_2 \pi^2\right)\alpha^4 -$$
$$\frac{1}{2}\left((a_0 + a_2)\pi^4 - R_I a_0 \pi^2 - R_I a_2 \pi^2 + \frac{30 Ta (R_I - \pi^2)}{\eta_1^2 (10a_2 + 2a_4 - 15a_0)}\right) = 0, \tag{19}$$

## 4.0 Results and Analysis

The heat source/sink, variable viscosity, and rotation impact on stability of Rayleigh-Bénard convection is demonstrated for space-dependent, variable, uniform heat sources. The effect of heat source is represented by the internal Rayleigh number ($R_I$), rotation as Taylor number ($T_a$), and the temperature-dependent viscosity as thermorheological parameter (V). The thermal Rayleigh number is the eigenvalue of problem. Some of the important highlights of linear stability theory are as follows:

1. Derivation of useful Fourier cosine series for base viscosity and temperature gradient.
2. Finding an analytical expression for the thermal Rayleigh number for all types of heat sources, which is necessary for working with stability theory.
3. Tabulation of variations of critical Rayleigh number and wavenumber with respect to the relevant parameter.

Here we have a tendency to shall study the impact of Space dependent heat supply on the linear stability of Rayleigh Bénard convection in temperature-sensitive Newtonian liquids with the heat source/sink and therefore the force. within the momentum equation, the term $Q_v$ indicates the warmth strength variation on the *y*-axis. To know the results arrived within the drawback higher we have a tendency to analyze the essential state temperature distribution, that throws lightweight on the discovered result of the heat supply (sink) on the steadiness. The scaled dimensionless temperature distribution for a space dependent heat supply is given by

$$\theta_b(Y) = \frac{T_b(Y) - T_0}{\Delta T} = 1 - Y. \tag{20}$$

**Table 1.** Table of critical Rayleigh and wave number for variable heat source

| | **$R_I = 0$** | | | |
|---|---|---|---|---|
| | **V = 0** | | **V = 0.5** | |
| | $R_{Ec}$ | $\alpha_c$ | $R_{Ec}$ | $\alpha_c$ |
| $T_a = 0$ | 719.46 | 0.7184 | 576.86 | 0.7185 |
| $T_a = 10$ | 740.189 | 0.7340 | 602.423 | 0.7422 |
| $T_a = 100$ | 897.925 | 0.8373 | 785.29 | 0.8828 |
| | $R_I = 0$ | | | |
| $T_a = 0$ | 657.51 | 0.7071 | 518.50 | 0.7085 |
| $T_a = 10$ | 677.071 | 0.7226 | 543.053 | 0.7330 |
| $T_a = 100$ | 826.282 | 0.8255 | 718.245 | 0.8765 |
| | $R_I = 0$ | | | |
| $T_a = 0$ | 597.309 | 0.6944 | 461.684 | 0.6975 |
| $T_a = 10$ | 615.732 | 0.7099 | 485.319 | 0.7229 |
| $T_a = 100$ | 756.517 | 0.8124 | 653.302 | 0.8706 |

Clearly, the basic temperature distribution is linear and varies with respect to $Y$- axis and does not depend on any parameters.

Figure 2 shows the effects of $R_I$, $T_a$ and $V R_I$ on vital Rayleigh and waveband. It is revealed that increasing the Taylor range lowers the critical Rayleigh number and

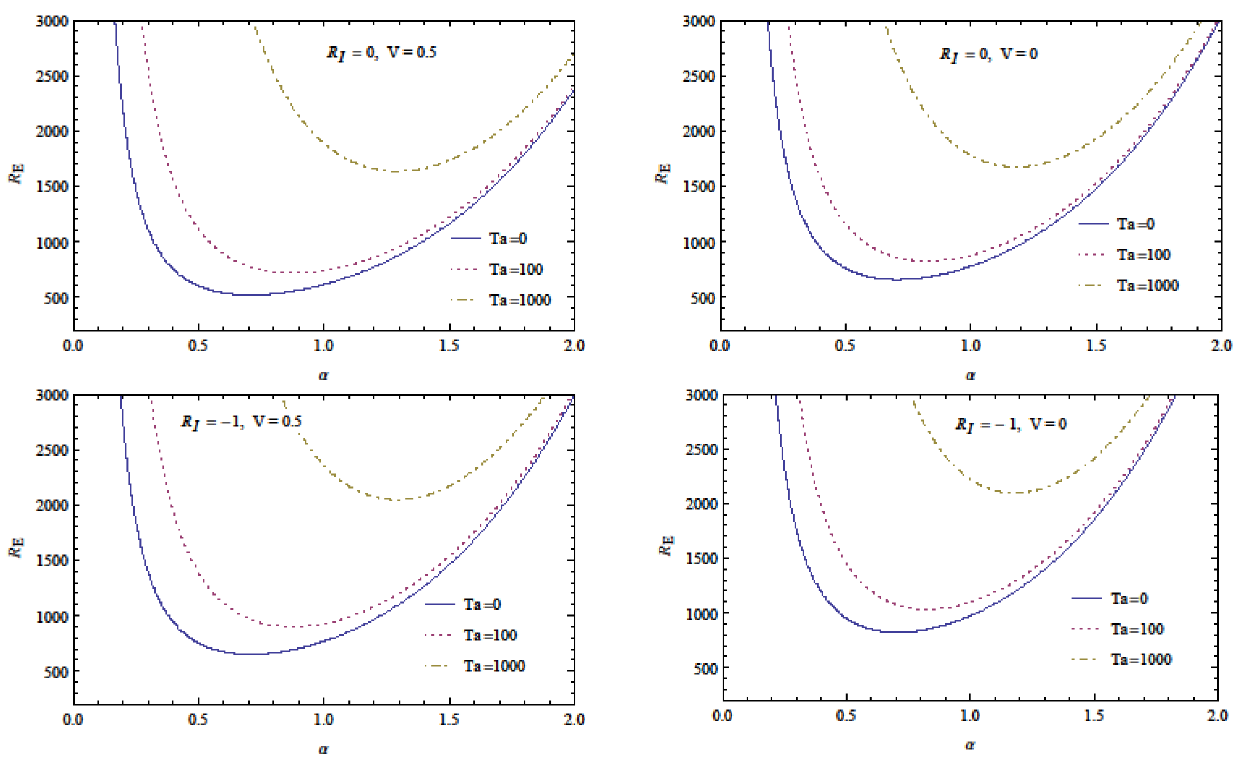


**Figure 2.** Plots Rayleigh number $R_E$ versus wave number $\alpha$ in the presence of space-dependent heat source

wavenumbers ($R_E$ and $\alpha_C$). Its action is to stabilize the device and delay the onset of convection. As $V$ increases, the important Rayleigh range and wave diversity decrease. This has the effect of destabilizing the device and promoting the onset of convection. When dealing with internal Rayleigh manifolds, the lower values of $R_I$ talk about heat sinking or absorption. From these plots we can see that the increase in large Rayleigh manifolds and wavebands is followed by an increase in the internal Rayleigh manifold $R_I$ and vice versa. Therefore, the effect of growing a large internal Rayleigh number is to destabilize the device, thereby promoting the initiation of convection and vice versa.

## 5.0 Conclusion

The paper reveals following important results

1. To conduct linear theory analysis, a truncated Fourier series representation of the basic state of temperature distribution and viscosity change in fuels is satisfactory.

2. The impact of increasing the internal Rayleigh number is to enhance the onset of convection, thereby accelerating the model all systems to be unstable system

3. The impact of increasing the variable viscosity parameter is to enhance the onset of convection within fuel, thereby accelerating the model all systems to be unstable system

4. The impact of increasing the strength of rotation is to delay the beginning of convection and it stabilizes the system for all type of fuels

5. It is possible to control the onset of convection by adjusting the parameters $R_I$, $V$ and $T_a$ appropriately.

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