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Couple Stress Fluid Flow Through a Porous Media Past a Solid Sphere

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Abstract

A steady, two-dimensional, incompressible couple stress fluid flow over a rigid sphere of radius 'a' surrounded by infinite porous region specifying a constant velocity away from the boundary is considered. An exact solution is found for the governing equations which leads to the expression for the stream function and shearing stress. The impact of couple stress parameter and porosity on the flow patterns is examined through streamlines. Also shear stress is computed for various values of couple-stress parameter and porous parameter. The obtained results reveal that as coupling stress parameter increases for fixing the porosity, streamlines are symmetric and meandered near the rigid sphere. But for fixed coupling stress parameter and increase in porous parameter cause the streamlines to move away from the solid sphere. Also, the dimensionless shear stress increases as couple-stress parameter intensifies for fixed porous parameter and vanishes at two stagnation points. The amplitude of the shearing stress raises with raise in porous parameter for fixed coupling stress parameter.

Keywords: *Continuity Equation, Couple Stress Parameter, Porosity*

1.0 Introduction

One of the important applications of porous media is combustion technology. Fuel Combustion is a chemical process in which a substance reacts rapidly with oxygen and releases heat. Thermal energy produced by burning either fossil fuels such as coal or oil or from renewable fuels such as firewood is collected for various purposes such as cooking, electricity generation or industrial or domestic heating. Combustion is also currently the only reaction used to power rockets. In oxy-fuel combustion, the fuel is burned in pure oxygen rather than in air. This technology recycles flue gas back into the furnace to regulate the temperature and replenish the volume of missing N_2 to ensure there is enough gas to maintain the temperature and heat flow profiles in the boiler. The inadequacy of the classical continuum method of describing composite fluids has led to the growth of micro continua theory. The original formulation of the general theory of micro continua have been disclosed[1] where fluid mechanism with deformable microelements is considered. The basic idea in conservation of linear and angular momentum is that there is a mismatch between them. When there is a match, however, the equations can be uncoupled because the term involving the spin can be obtained in terms of velocity. Such fluids are called couple-stress fluids. The theory of couple-stress fluids which is a subclass of micropolar fluid was developed[2].

For many applications of engineering and Bio-mechanics, there is a need to understand the couple stress fluid flow behavior past a sphere and cylinder. The study of Stokes flow of couple stress fluid over a circular shell made up of pair of two con focal spheroids S_0 and S_1 such that S_0 was placed inside S_1 was carried out[3]. The inner section of S_0 had a couple stress fluid, an area that separates S_0 and S_1 was considered to be porous area. Using the Stokes

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equation, obtained the solution analytically and also the expressions for flow variables according to Legendre functions, radial and angular prolate spheroidal wave functions. Shell pressure was measured and drag was calculated. Numerically studied the impact of porosity, geometric parameter and couple stress parameter on the drag and presented graphically. It has been observed that, drag reduces as eccentricity of the outer spheroid increases for fixed S_0. The complete solution for the basic Couette flow, Poiseuille flow and the flow of standard Couette flow for incompressible couple stress fluid between equivalent plates was obtained[4]. They found that when the bottom plate moves to the top plate the Couette flow velocity increases remarkably and boundary velocity declines for more the fluid slides at the boundary.

The slow flow of a solid slippery sphere surrounded by couple stress fluid was examined[5]. A simple formulation for drag which acts on solid sphere was obtained. The special case of obtained drag formula was determined and equated with the corresponding results of the previous works. The normal drag force on the sphere was gained and presented by the graphs and discussed the impact of slip parameter and coefficients of viscosity. The steady flow caused by the rotation of a porous sphere having thin surface on its symmetric axis in incompressible couple stress fluid at rest was studied[6]. They examined the effect of couple stress parameter on the porous sphere. The smaller the couple stress parameters, the more particles of fluid are discarded from the sphere. Also, inner flow was identical to the viscous flow and because of this inner flow the couple experienced by the sphere was zero. This study was different from that of the micro polar fluid. The flow of incompressible couple stress fluid on a rotary oscillation of a solid sphere was analyzed[7]. No slip boundary condition was applied at solid sphere surface. In addition, vanishing couple stress condition was considered on the sphere boundary. In this study, movement is produced by a sudden rotation of a solid sphere about an axis that moves through its midpoint at a time-dependent velocity. The torque acting on spherical body due to couple stress fluid was obtained using an integral formula and found that η has a small effect on torque. For viscosity parameter η as zero, the old Navier-Stokes theory can be found.

The analytical study of steady flow of incompressible couple stress fluid past a solid sphere surrounded by permeable area was carried out[8]. The drag force was calculated inside the porous area exerted by considered fluid on the sphere and the same is illustrated in graphs. The translation motion of quasi-steady incompressible couple stress fluid between two eccentric circular sphere surfaces has been investigated[9]. The principle of superposition was used to obtain a general solution for governing the equations using two spherical systems associated with the two spherical centers. To meet the requirements of considered boundary conditions at spherical surfaces, a numerical boundary collocation technique was used. The drag was obtained on spherical particles and presented graphically. The current numerical results have shown that drag force increase equally with the increase in radius. It also increases when the viscosity parameter and couple stress parameter increases. The oscillating incompressible couple stress fluid flow around porous sphere has been studied[10]. The effects of geometric parameters, couple stress parameter and permeability on the considered flow is presented through graphs. They made a conclusion that drag of viscous fluid flow is less than that of couple stress fluids. An attempt was made to study an oscillating incompressible couple stress fluid caused by oscillations of two concentric spheres[11]. The concentric region in between the spheres were filled with couple stress fluid and assumed that oscillations occur about a diameter maintaining the constant frequency and different angular velocity. The stress tensor and couple stress tensor components are obtained. The model of two-dimensional incompressible dissipative free convection couple stress fluid flow over a cylinder in the presence of magnetic force, thermal radiation, porous medium and chemical reaction effect was presented[12]. The impact of these parameters on velocity, temperature and concentration of considered flow were studied. They observed that intensifying the magnetic number, decreases the velocity and amplifies the profiles of concentration. Also increase in buoyancy ratio parameter magnifies the velocity profile.

For many applications of engineering and bio-mechanics, there is a need to understand the fluid flow around a sphere or cylinder. In this chapter, the couple – stress fluid flow past a rigid sphere of radius 'a' surrounded by porous sphere of radius '$a<\infty$' specifying constant velocity away from the boundary is considered. The flow is assumed to be axi-symmetric. An analytical solution has been obtained for the governing equations. A closed form exact solution has been found analytically. Streamlines are obtained and shear stress has been computed for distinct values of couple stress parameter and porosity.

2.0 Mathematical Formulation

Here steady, incompressible couple stress fluid flow around a rigid sphere of radius 'a' bounded by porous region of radius 'a<∞' is considered as shown in the Figure 1. The flow is assumed to be two dimensional with uniform velocity u_∞.

The governing equations are as below:

Continuity equation,

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

Momentum equation,

$$\nabla p = \mu \nabla^2 \vec{q} - \eta' \nabla^4 \vec{q} - \frac{\mu}{k} \vec{q} \tag{2}$$

To get dimensionless governing equations, the following non-dimensional parameters are used:

$$p^* = \frac{ap}{\mu u_\infty}, \vec{q}^* = \frac{\vec{q}}{u_\infty}, \nabla^* = \nabla a \tag{3}$$

The governing equation of couple stress tensor is given by

$$m_{ij} = m\delta_{ij} + 4(\eta \omega_{j,i} + \eta' \omega_{i,j}) \tag{4}$$

For the axi-symmetric flow, the velocity components in terms of stream function are defined as,

$$u = \frac{1}{r^2 \sin\theta} \frac{\partial \psi}{\partial \theta} \;\; ; \;\; v = \frac{-1}{r \sin\theta} \frac{\partial \psi}{\partial r} \tag{5}$$

Removing the pressure term from equation (2) by taking curl on both the sides, we get sixth order partial differential equation in the form:

$$E^6 \psi - \lambda^2 E^4 \psi + \sigma^2 \lambda^2 E^2 \psi = 0 \tag{6}$$

Let us assume $\psi = \psi_0 + \psi_1 + \psi_2$, using this resolution an equation (6) becomes

$$E^2 \psi_0 = 0 \tag{7}$$

$$(E^2 - \xi_1^2)\psi_1 = 0 \tag{8}$$

$$(E^2 - \xi_2^2)\psi_2 = 0 \tag{9}$$

where

$$\xi_1^2 = \frac{\lambda^2 + \lambda\sqrt{\lambda^2 - 4\sigma^2}}{2}, \quad \xi_2^2 = \frac{\lambda^2 - \lambda\sqrt{\lambda^2 - 4\sigma^2}}{2}$$

To solve these equations, we use the following boundary conditions.

Impenetrability condition at the surface of the solid sphere:

$$u(a,\theta)=0 \tag{10}$$

$$v(a,\theta)=0 \tag{11}$$

The vanishing couple stress condition:

$$m_{r\varphi}=0 \tag{12}$$

where

$$m_{r\varphi} = -r^{-3/2}\left[\left((2\eta + \eta')\xi_1^2 K_{3/2}(\xi_1 r) + \eta \xi_1^3 K_{1/2}(\xi_1 r)\right)B + \left((2\eta + \eta')\xi_2^2 K_{3/2}(\xi_2 r) + \eta \xi_2^3 K_{1/2}(\xi_2 r)\right)C\right] \sin\theta$$

is the couple stress component

Further, the stream function for uniform flow far away from the bondary is:

$$\psi(r,\theta) \sim \frac{r^2}{2} \sin^2\theta \quad as\ r \to \infty \tag{13}$$

3.0 Method of Solution

From the equation (13), one should look for similarity solution of equations (7) to (9) in the form

$$\psi(r,\theta) = f(r) \sin^2\theta \tag{14}$$

Substituting the equation (14) in equations (7) to (9), we obtain corresponding second order ordinary differential equation and then using the transformation function

$$f(r) = \sqrt{r}\, S(r) \tag{15}$$

The obtained general solution is given by,

$$f(r) = r^2 + \frac{A}{r} + B\sqrt{r}\, K_{3/2}(\xi_1 r) + C\sqrt{r}\, I_{3/2}(\xi_2 r) \tag{16}$$

Hence the solution in terms of stream function is given by

$$\psi(r,\theta) = \left(r^2 + \frac{A}{r} + B\sqrt{r}\, K_{3/2}(\xi_1 r) + C\sqrt{r}\, I_{3/2}(\xi_2 r)\right) sin^2\theta \tag{17}$$

where A, B and C are arbitrary constants which can be evaluated with help of boundary conditions from the equations (10) to (12).

4.0 Results and Discussions

We examined the effect of porosity and couple-stress parameter on the flow of couple-stress fluid around a solid sphere under the assumptions and boundary conditions mentioned above. The flow characteristics are analyzed for various values of dimensionless parameters.

The flow patterns are observed by varying couple stress parameter for fixed porosity σ=0.5 and the results are shown in Figures 2 (a) to 2 (d). From figures it is noticed that the flow is symmetric behind and front of the sphere and no separation occurs near the stagnation point. Also, the streamlines are denser near the wall because of current effects. Similarly for the fixed couple stress parameter λ=0.5, the streamlines can be seen for σ=2. Once the porosity is raised to σ=4, there is a slight decrease in permeability of the porous region. Thus, the streamlines are initiated to move away from the solid core.

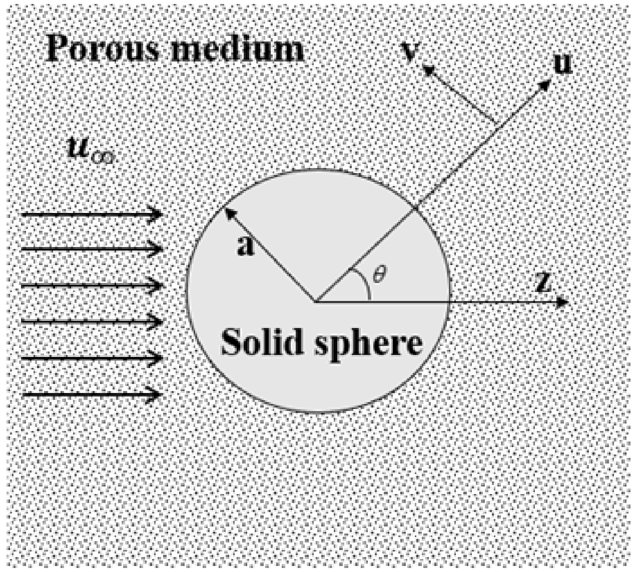
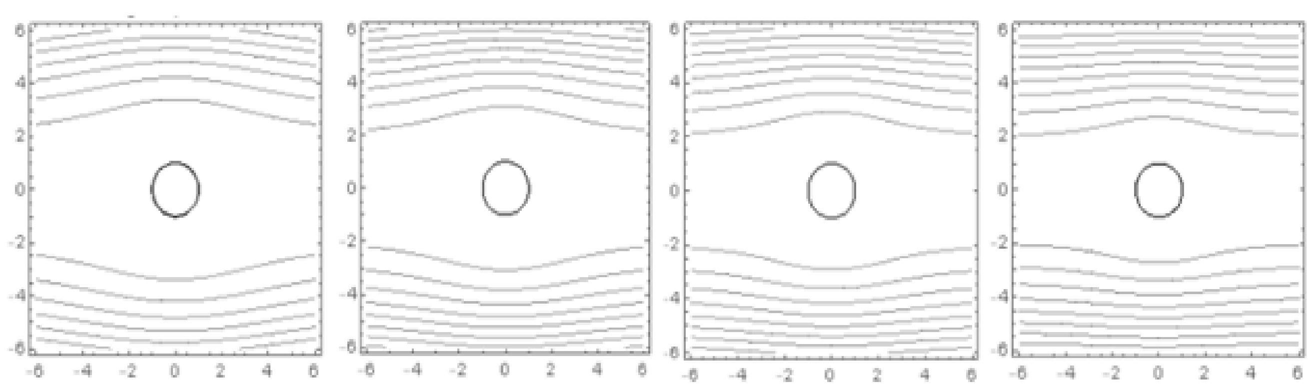


Figure 1. Physical Configuration

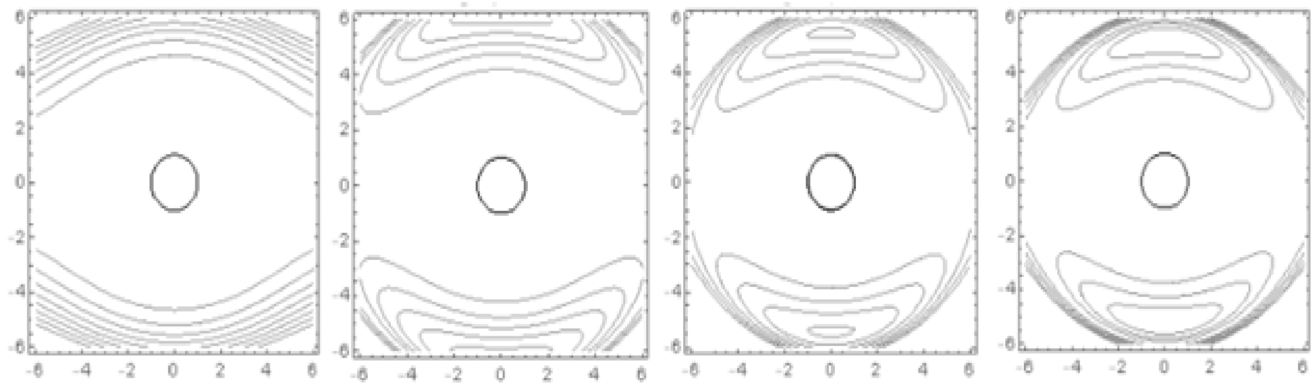
Further magnifying the porosity to σ=5 & 6, decreases the fluid flow in porous section. Hence the streamlines are away from the solid sphere is noticed (See Figures 3 (a) to 3 (d)).

The variation of dimensionless shear stress $\tau_{r\theta}$ on the surface of the sphere is presented at r=1 with θ for several values of couple stress parameter and porous parameter. It is observed that for fixed porosity σ=0.5, increase in couple stress parameter increases the shear stress at any point on the surface of the sphere. At the forward



(a) λ=0.5 (b) λ=1 (c) λ=1.5 (d) λ=2

Figure 2. Streamlines for fixed σ=0.5 for different value of λ



(a) σ=2 (b) σ=4 (c) σ=5 (d) σ=6

Figure 3. Streamlines for fixed λ=0.5 for different value of σ

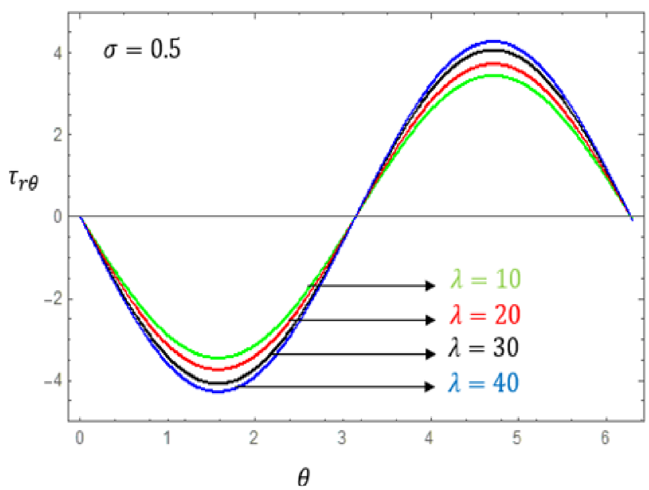


Figure 4. Variation of shearing stress by varying couple stress parameter for fixed porosity σ

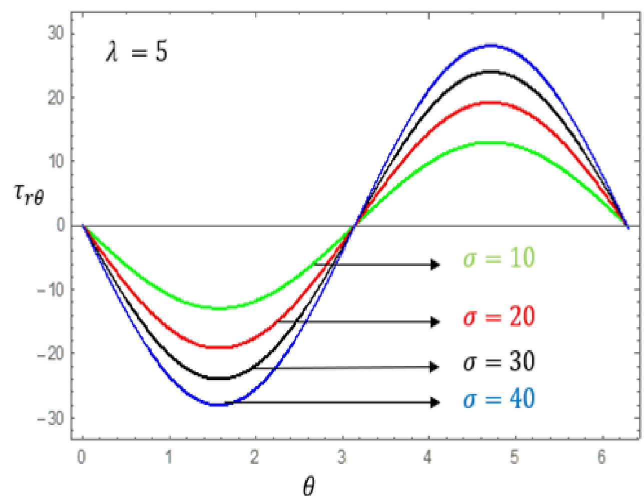


Figure 5. Variation of shearing stress by varying porosity for fixed couple stress parameter.

stagnation point i.e., at θ=0 and θ=π the shear stress vanishes whereas it attains a maximum value at θ=±π⁄2 and the same is witnessed in the Figure 4. Similarly varying the porosity for fixed couple stress parameter λ=5 shear stress is periodic in nature, reaches its maximum value for θ=±π⁄2 and vanishes at the end points as shown in Figure 5.

5.0 Conclusions

This chapter presents the steady, incompressible, viscous and couple stress fluid flow past an impermeable sphere for uniform velocity away from the boundary. The similarity solution method is adopted to solve the physical problem. The effect of dimensionless parameters involved in the problem on the streamlines and shearing stress are discussed.

The streamlines are symmetric and denser near the solid sphere for increase in coupling stress parameter by fixing porosity. But for the fixed couple stress parameter and for increase in porous parameter cause the streamlines to move away from the solid sphere due to decrease in permeability of the porous region. Also, the dimensionless shear stress increases for increase in couple stress parameter by fixing porosity and vanishes at two stagnation points.

6.0 Acknowledgement

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