

A Nanofluid Boundary Layer Flow Over a Stretching Body in the Presence of Porous Medium

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Abstract

A numerical approach is presented to investigate fluid concentration, velocity and temperature like flow characteristics for the nano fluid boundary layer flow over a stretching sheet in the presence porous medium. The equations of ordinary differentiation are obtained from the governing equations of partial differentiation using similarity transformations. The reduced equations of ordinary differentiation have solved numerically using both Spectral Collocation Method and Shooting Technique which involves R-K Fehlberg and N-R method. The computation results are drawn for dimensionless parameters like Lewis number, Brownian motion parameter, thermophoresis parameter, thermal diffusivity, Prandtl number and porous parameter on the fluid flow, concentration and temperature characteristics. The computation results dictates that the enhancement of the porous parameter diminishes the flow whereas temperature and concentration enhance in this region. Due to nanofluid there is a rise in thermal conductivity of fluid flow. The polymer drawing and extrusion, casting, hot rolling, metal cooling and lot of engineering processes made use of the computational results and information understood very well.

Keywords: Brownian Motion Parameter, Permeability Parameter, Porous Media, Spectral Collocation Method, Stretching Sheet, Thermophoresis Parameter

1.0 Introduction

A colloidal solid, particles of 10-1000nm made of macromolecular materials contain encapsulated active ingredient have been defined as nano particles. Nanoparticles are found in volcanic ash, fine sand and dust, biological matter, human industrial and domestic activities. A smaller amount of nanosized particles which are uniform and stable are suspended in a liquid gave rise

to new type of heat transfer fluids namely nanofluids. Dispersion of these particles intensifies nanofluids thermal conductivity and convective heat transfer rates remarkably. Because of its widespread application in engineering and industry, researchers place a high value on flow adjacent to a stretching sheet. These models find extensive usage in the fabrication of lengthy and uniform metal components, rubber sheets and corrosion-resistant fabrics, metalworking procedures like hot rolling, and

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the creation of glass fiber for thermal insulation and emollients.

Porous surface is a solid matrix having enough open space in and around solids which enables a fluid to pass through it. Porosity is a measure of volumetric fraction of pores in the medium. Some porous media occur naturally as limestone, beaches and human lungs while some are man-made like cement, ceramics and sponges. Porous medium has wide applications in industries and engineering which have captivated many researchers. To name a few are textile industries to wither porous materials, in the process of purification and filtration, drying of paint, mining of crude oil and in the manufacture of toughened glass. For thermal and economical systems to operate more efficiently, boosting the rate of heat transfer is crucial. Techniques for enhancing the rate of heat transmission through the use of nanofluid and porous media have been proposed for this aim.

Darcy law assumes that the continuous flow depends on pressure gradient and force due to gravity. A densely packed porous medium with lower permeability is considered as Darcy model but in case of Darcy-Brinkman model where the porous medium is sparsely packed shear velocity along with Darcy term is considered and it has high permeability. The region of a larger flow that is adjacent to the surface having effects of frictional forces near wall is referred to as boundary layer flow. The hot rolling, glass blowing and many more engineering works made use of boundary layer flow over a stretching body.

Spectral methods are known for their accuracy and exactness unlike old stable methods like finite difference methods. It reduces time consumption, increases rate of convergence, requires few grid points, simple to implement and gives accurate solution on differential equation, as a reason for which it is applied in other fields of sciences. Owing to such exceptional characteristics, they have gained immense interest by researchers in multidisciplinary fields such as cooling of electronic articles, heat exchanger, solar energy, oil recovery, geothermal sites and so on. Shooting method is useful and efficient in solving discrete boundary condition problem and this is reduced to initial condition problem by making some initial guesses.

Khan and Pop laid foundation for nano fluid flow study over a stretching body¹. Gorla and Chamkha

analyzed free convection of saturated nano fluid flow embedded in porous channel over a horizontal plate². The Jeffrey monitored nano fluid onto a stretching body under the effect of elastic variable, thermophoresis and Brownian motion³. Chamkha and Aly studied Soret and Dufour numbers of micro polar fluid about stagnation point throughout a stretching surface in a porous duct⁴.

Pal and Mandal⁵ and Krishnamurthy *et al.*,²⁵ observed the melting, radiation and chemical effects on Williamson fluid towards continuously stretching body in porous outlet using Shooting technique which involves R-K Fehlberg method. The flow pattern, mechanism of heat transfer and various models adopted to explain the characterization of porous medium was discussed⁶. Costa spelled out the foundations and accuracy of spectral methods over finite difference method⁷. Malik *et al.*, solved Navier's stokes equation by Chebyshev spectral method which can be applied to any boundary layer with variable viscosity⁸.

Kasaean *et al.*, reviewed latest development in nanofluid flow placed in a porous medium followed by enhanced heat transfer rates both nanofluids and porous medium in the system⁹. Abbas bandy and Shivanian gave a mechanism based on pseudo-spectral technique to simplify dual solution of steady mixed convection flow in porous medium at infinite condition¹⁰. Wright solved ordinary differential equation by finite Chebyshev series by considering selected points for initial value problem¹¹. Comparative study between successive linearization method and computational software MATLAB for non-linear initial value problem was done by Motsa¹² and Bhatti *et al.*,¹⁴.

Kumar *et al.*, explained the hold of second order slip mixed convection, micro rotation, viscous dissipation, Brownian motion, chemical reaction on fixed 2-D stagnation point flow of a polar fluid in a porous vent due to a stretching counter¹³. Magagula *et al.*,¹⁵ and Motsa²¹ analyzed flow of incompressible magnetohydrodynamic fluid and showed that the computation time to obtain the solution is very less using Bivariate spectral quasilinearization method compared to long established methods. Motsa¹⁶ developed new algorithm to solve boundary value problem which was more accurate, stable and efficient in comparison with other prevailing methods.

Non-linear equations of partial differentiation are solved using spectral methods in space combined with finite difference in time¹⁷. Alharbey *et al.*,¹⁸ showed the hold of Reynold's number and Brinkman number on entropy generation is a key feature to the flow. Kameswaran *et al.*,¹⁹ discussed the analysis of mass and heat transfer in different nanofluids placed in saturated porous passage across a stretching area by novel spectral collocation method. Flows over wedge have scope in engineering and physical application like greasing, cooling of inclined surfaces etc.

Gogo *et al.*,²⁰ explained thermodynamic irreversibilities. Flow and fluid specification on velocity and temperature profile, entropy generation are noticed on a nanofluid in the influence of porous wedge. Vijaykumar *et al.*, studied heat and fluid movement inside porous materials using a macroscopic approach²⁶. Mahantesh studied the thermal attributes of a Casson nano fluid in Darcy porous media over an exponential stretching surface²². Non -Newtonian nano fluid has lower concentration and temperature and higher velocity than classical nano fluid²³. Vijayakumar *et al.*, studied thermal control in porous enclosures to promote natural convection²⁴.

The objective of communicating this research work is to discuss 2-D Navier-Stokes equation for Darcy-Brinkman flow created by a stretching sheet with viscous and porous dissipation. With very minute changes this conclusion can be applied to curved channel flow, Couette flow particularly cylindrical and boundary layers which are parallel. The governing non-linear equations of partial differentiation have transformed into equations of ordinary differentiation and solved Spectral Collocation technique and Shooting technique which involves R-K Fehlberg and N-R method and both have employed to check the results accuracy in the current study. The variation of relevant constants on velocity, concentration and temperature are interpreted and analyzed. The findings of the current study are to facilitate other research and application.

2.0 Mathematical Formulation

The geometrical interpretation of the problem is as presented in Figure 1. A nano fluids steady 2-D boundary layer flow onto a stretching body in presence of porous

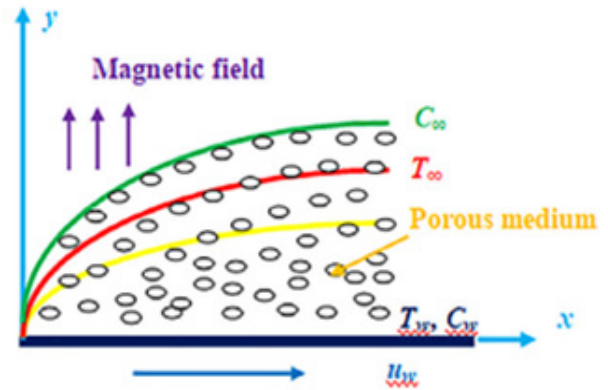


Figure 1. Geometrical interpretation.

medium has been considered. The plate is elongated along x -axis with velocity ' ax '. Let U_w , T_w and C_w are the velocity of the fluid, temperature and concentration of nanoparticle at the wall. The temperature (T) and concentration of nanoparticle (C) of the fluid in free stream are and With usual assumptions and notations of boundary layer flow, the basic conservation equation of mass, momentum, energy and concentration have presented in expressions (1)-(5).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_f} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\bar{\mu}}{k\rho_f} u \tag{2}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_f} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\bar{\mu}}{k\rho_f} v \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \tau \left\{ D_B \left(\frac{\partial C}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + \left(\frac{D_T}{T_\infty} \right) \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right] \right\} \tag{4}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \left(\frac{D_T}{T_\infty} \right) \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{5}$$

Here u and v are velocities in x and y directions respectively, p is the fluid pressure, density and nanoparticle density are denoted as ρ_f and ρ_p respectively. Let fluid kinematic thickness be ν , thermal diffusivity

be α , thermophoresis diffusion co-efficient be D_T , D_B be Brownian diffusion co-efficient, effective heat capacity be $\tau = (\rho c)_p / (\rho c)_f$ with heat capacity of nanoparticles as $(\rho c)_p$ and nano fluid heat capacity as $(\rho c)_f$ and thermal conductivity be k .

Using the following boundary conditions

$$v = 0, u = U_w(x) = ax, T = T_w, C = C_w \text{ at } y = 0$$

$$u = 0, v = 0, T = T_\infty, C = C_\infty \text{ as } y = \infty \quad (6)$$

and by introducing the following transformation variables

$$\Psi = (av)^{1/2} x f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \eta = (\alpha|v)^{1/2} y \quad (7)$$

η represents similarity variable, velocity component u and v are defined in terms of stream function Ψ as $u = \partial\Psi/\partial y$ and $v = -\partial\Psi/\partial x$.

$$f''' + f f'' - f'^2 - k_p f' = 0 \quad (8)$$

$$\frac{1}{Pr} \theta'' + f \theta' + Nb \phi' \theta' + Nt \theta'^2 = 0 \quad (9)$$

$$\phi'' + Le f \phi' + \frac{Nt}{Nb} \theta'' = 0 \quad (10)$$

subjected to

$$\begin{aligned} f(0) = 0, f'(0) = 1, \theta(0) = 1, \phi(0) = 1 \\ f(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \end{aligned} \quad (11)$$

Here prime represents differentiation with respect to η

$$Pr = \frac{v}{\alpha} \text{ is Prandtl number,}$$

$$Le = \frac{v}{D_B} \text{ is Lewis number,}$$

$Nb = \frac{(\rho C)_p D_B (\phi_w - \phi_\infty)}{(\rho C)_f v}$ denotes Brownian motion parameter,

$Nt = \frac{(\rho C)_p D_T (T_w - T_\infty)}{(\rho C)_f T_\infty v}$ stands for thermophoresis parameter,

$$k_p = \frac{\bar{\mu}}{k \rho_f a} \text{ spells permeability parameter.}$$

The analytical solution of Eq (8) is given by

$$f(\eta) = \frac{1}{s} (1 - e^{-s\eta})$$

$$\text{where } s = \sqrt{1 + k_p} \quad (12)$$

Using the non- dimensional variables,

$$Nu = \frac{x q_w}{k(T_w - T_\infty)} \text{ as Nusselt number,}$$

$$Sh = \frac{x q_m}{D_B(C_w - C_\infty)} \text{ denoting Sherwood number,}$$

$Re_x = U_w(x)x/v$ as Reynolds number where q_w and q_m are local heat and mass flux respectively, we obtain

$$Nur = Re_x^{-1/2} Nu = -\theta'(0), Shr = Re_x^{-1/2} Sh = -\phi'(0) \quad (13)$$

With Nur and Shr respectively denoting reduced Nusselt number and reduced Sherwood number at the surface.

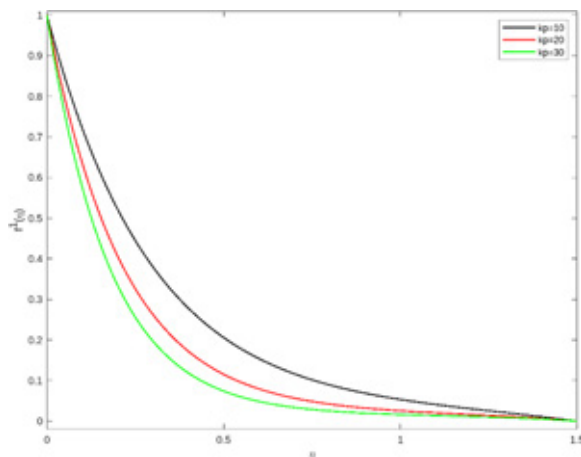
We presented a Quasi-linear Spectral Collocation Method having transformation $\eta = \frac{L}{2} (z + 1)$ which converts physical domain to computational domain with Gauss-Lobatto points given by $z_j = \cos\left(\frac{\pi j}{N}\right)$ $j = 0(1)N$. Using pseudo inverse operator in MATLAB the resultant system of equations are solved. The linear and nonlinear equations of ordinary differentiation subjected to boundary conditions can be handled with the algorithms presented in this work.

In the Shooting Technique any boundary value problem can be transformed into an initial value problem. The estimated boundary values must match the actual boundary values. The boundary value problem can be approached closer to feasible with the help of scientific methodology or trial and error experimentation. The selection of the right finite value for the far field boundary condition is the most vital phase in this approach. At a large but finite value of, where there are no appreciable changes in velocity, temperature, and other variables,

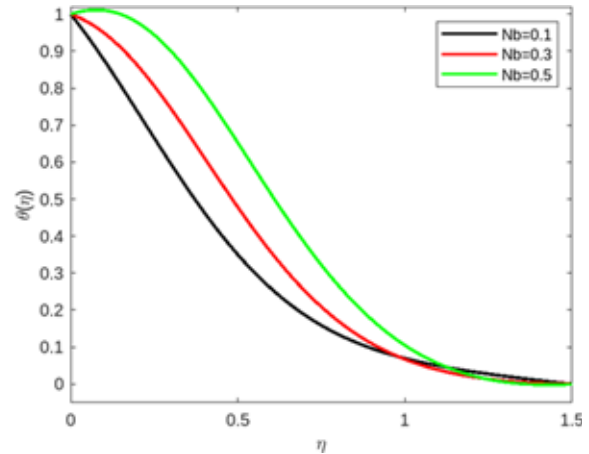
we took the infinity condition. The analogy shows good harmony for each values considered.

3.0 Results

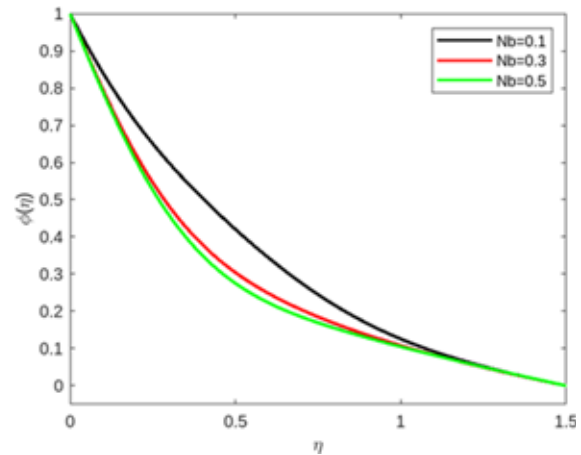
The equations of partial differentiation presented through (8) - (10) with boundary conditions (11) are reduced to equations of ordinary differentiation and solved using both spectral collocation technique and shooting technique which involves R-K Fehlberg and N-R method. To monitor the exactness in the numerical methods applied, the outcomes are contrasted with the accessible sources of Khan and Pop¹. It is observed that the values obtained by Spectral Collocation Method are more accurate than Shooting Technique in the Table 1. Graphs are plotted with respect to the values obtained to look over the impact on permeability, velocity, Brownian



Graph 1. Behaviour of k_p on velocity profile



Graph 2. Behaviour of Nb on temperature attributes.

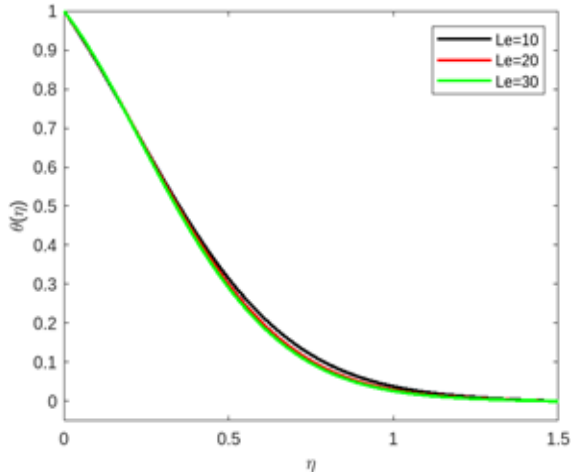


Graph 3. Behaviour of Nb on concentration attributes.

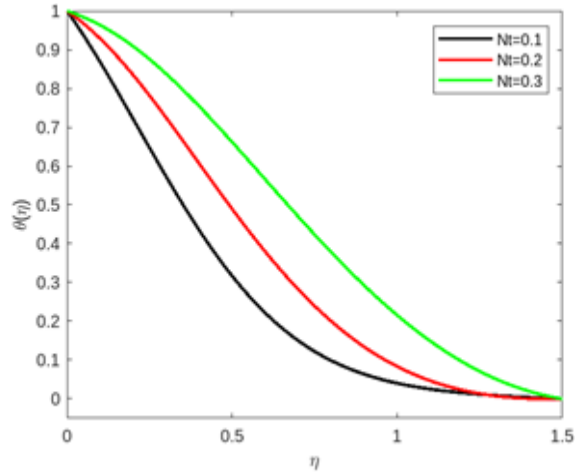
motion, Prandtl number, thermophoresis parameter and Lewis number.

Table 1. Comparison of present study *Nur* and *Shr* with results of Khan and Pop¹⁹

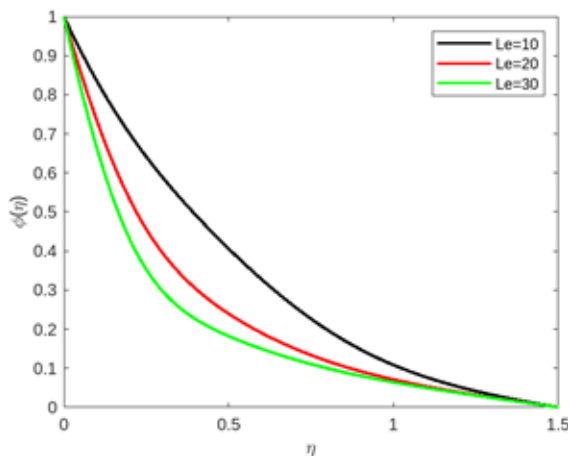
Nt	Nur			Shr		
	W. A. Khan and I. Pop ¹⁹	Spectral Collocation Technique	Shooting Technique	W. A. Khan and I. Pop ¹⁹	Spectral Collocation Technique	Shooting Technique
0.1	0.9524	0.9522	0.9520	2.1294	2.1294	2.1292
0.2	0.6932	0.6934	0.6928	2.2740	2.2742	2.2737
0.3	0.5201	0.5203	0.5197	2.5286	2.5287	2.5283
0.4	0.4026	0.4028	0.4022	2.7952	2.7953	2.7949
0.5	0.3211	0.3213	0.3207	3.0351	3.0353	3.0348



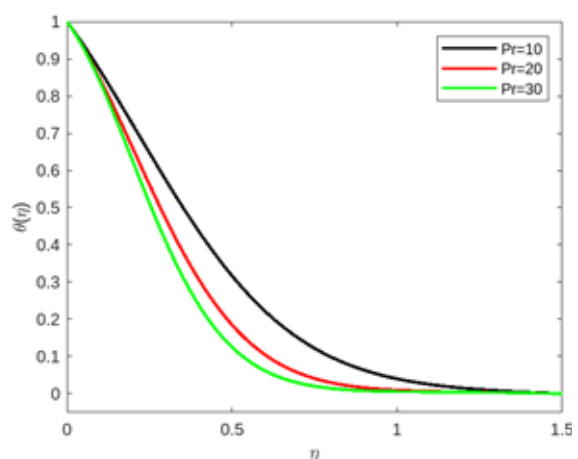
Graph 4. Behaviour of *Le* on-temperature attributes.



Graph 6. Behaviour of *Nt* on temperature distribution.



Graph 5. Behaviour of *Le* on-concentration attributes.



Graph 7. Behaviour of *Pr* on temperature profiles.

Graph 1 depicts the velocity profile corresponding to permeability parameter with $Pr=10$, $Le=10$ and $Nb=Nt=0.1$. There is a downtrend velocity profile with increased permeability parameter as observed in graph above. An increase in the viscosity of the nano fluid and porous medium permeability decrease led to a fall in thickness of velocity boundary layer.

Graph 2 and Graph 3 explain the response of Nb on temperature and nanoparticles concentration with $Pr=10$, $Le=10$, $k_p=0.9$, $Nt=0.1$. It is clear that the temperature rises and concentration decreases as Nb is increased. This is because of enhancement of heat transfer and disperse of nanoparticles. The Brownian motion is proportional to the temperature directly nanoparticles gain energy. It therefore increases, with increase in temperature.

The opposite behaviour on concentration against the Brownian motion parameter owing to the thinning of the boundary layer since random movement of fluid particles weakens concentration.

Graph 4 and Graph 5 portraits the behaviour of Le on temperature and concentration profile by taking $Pr=10$, $k_p=0.1$, $Nb=Nt=0.1$. It is very clear that both nanoparticle fraction and temperature decrease with increase in Le . The effect of Le on concentration is more significant than on the temperature. The boundary layer thickness of concentration decreases with rise in Lewis number since Brownian diffusion co-efficient D_b also decreases.

Graph 6 illustrates the behaviour of Nt on temperature distribution with $Pr=10$, $Le=10$, $Nb=0.1$, $k_p=0.1$. The temperature profiles enhance with the increase in Nt . The

fact that the thermophoretic force created by temperature potential also causes a flow fast away from stretching sheet. Accordingly, hotter fluid moves away from the stretching body. Hence thermal boundary layer thickness rises with rise in the value of Nt .

Graph 7 exhibits the influence of Pr on temperature field for having $Nb=Nt=0.1$, $Le=10$, $k_p=0.1$. A rise in Prandtl number Pr causes decrease in temperature inside boundary layer. The rise in fluid thermal conductivity causes reduction in heat transfer rate from the stretching sheet and thus decrease in thermal boundary layer.

4.0 Conclusions

The boundary layer flow over a stretching body placed in a porous medium been analyzed in this work. To convert conservative form of momentum, energy and nanoparticle concentration into equations of ordinary differentiation, the similarity transformations have utilized. The Spectral Collocation technique and Shooting Method were used to obtain the solution. The results of W. A. Khan and I Pop been referred to rise a comparison in this study on Sherwood number and Nusselt number in the absence of porous media¹⁹ and found very close. The variations in velocity, concentration and temperature have been presented for varied k_p , Pr , Le , Nt and Nb parameters. The following distinct conclusions are expressed from this experimental work;

- Velocity boundary layer thickness diminishes with the rise in permeability parameter k_p and stretching parameter η .
- As Nb increases with the increase in η , the temperature increases and concentration decreases respectively which is due to increased rate of heat transfer and random motion of fluid particles.
- Both temperature and concentration diminish with the rise in Le and the effect is seen more on concentration due to decrease in random motion of particles.
- The temperature rises with increasing values of Nt while a reverse behavior of temperature is found with increase in the value of Pr .
- This work helps us to investigate heat and fluid transport inside porous materials.

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