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Analysis of Forchheimer Effect for Double Diffusive Convection with Dusty Fluids and MHD

S. Kavitha[1]*, N. Nalinakshi[2], P. A. Dinesh[3] and Brijesh[4]

[1]*Department of Mathematics, Mount Carmel College, Bengaluru - 560052, Karnataka, India; kavi.sakthivel81@gmail.com*

[2]*Department of Basic Sciences, Atria I.T, Bengaluru - 560024, Karnataka, India; bsehod@atria.edu*

[3]*Department of Mathematics, Ramaiah Institute of Technology, Bengaluru - 560054, Karnataka, India; dineshdpa@msrit.edu*

[4]*Department of Chemical Engineering, Ramaiah Institute of Technology, Bengaluru - 560054, Karnataka, India; brijesh@msrit.edu*

Abstract

An attempt has been made to analyze the effect of second order resistance for a steady, dusty fluid considering magnetohydrodynamic (MHD) and also the characteristics of fluid like permeability, porosity, solutal diffusivity and thermal conductivity being varied. Here the basic equations are coupled, non-linear Partial Differential Equations (PDEs), which are changed by similarity transformations to higher order Ordinary Differential Equations (ODE). After being transformed the higher order ODE that where obtained are resolved numerically. Shooting technique is employed here and the values are tabulated for various pertinent parameter variations. The effects of the inertia, concentration and interaction, mixed convection, magnetic and many other parameters are discussed and plotted graphs for velocity, concentration and temperature. The magnetic force enhances velocity. It was found that the present study correlates with the existence results.

Keywords: *Dust Particles, Forchheimer, Mixed Convection, MHD, Variable Fluid Properties*

1.0 Introduction

The double dissipation has various application in the field of metallurgy, astrophysics, geology, oceanography, packed bed catalytic reactors are a few to mention. To understand the rate of heat transfer in a better way, Lai and Kulchi studied the variable viscosity in vertical plate[1]. The variable permeability and porosity has more influence on the boundary layer flow which was first analysed by Mohammadein *et al.*, numerically[2]. Chandrasekhara, B. C *et al.*, gave the criteria for double diffusion and mixed convection[3]. Also showed that the non-dimensional parameters like Richardson number controls the flow. Generally, in porous media, classical Darcy's law was used to model the flow patterns which is inadequate for higher velocity and porosity, the square of velocity term was included in the velocity equation by Forchheimer[4]. Nakayama showed that as the inertial term is increased, the temperature is also increased[5]. Andersson and Aarseth gave the generalized similarity transformation and revised Sakiadis flow[6]. Pal and Shivakumara found that when the second order resistance is increased the rate of cooling is faster[7]. Provoked by the applications and research Nalinakshi *et al.*, well-considered the study of a heated plate in porous media considering the inertial term[8]. The researchers considered the inertial term in

*Author for correspondence

the momentum equation and internal heat generation in thermal equation, resulted in showing faster heat transfer because of the Forchheimer term[9-10].

The incompressible fluid that conducts electricity influenced by the magnetic effect known as Magneto-hydrodynamic (MHD) plays a vital role in the treatment of asthma and tumor, X-ray radiation, gastric medicines etc. The cooling rate is regulated by the MHD influenced fluids. MHD generators, nuclear reactors, geothermal extraction energy are a few to mention in engineering practices. Aldoss *et al.*,[11]. gave a theoretical analysis of MHD in a heated vertical plate. Elbashbeshy[12] illustrated the effect of MHD with temperature and concentration being not constant. Elgazery and Hassan[13] analyzed the effect of MHD and fluid properties varying in stretching sheet. Nalinakshi *et al.*,[14] stated the effect of Forchheimer in the presence of magnetic fields which has an influence in the field of fluid flow.

The impurities or small particles slinged in fluid known as dusty fluid actually reduce the efficiency of any machine but they are constructive in many industrial process like polymer, paper. The waste coal dust from coal mining industry can be added 3% of calcareous clay to get 60% of thermos block. They are also important in nursing waste water, Saffman[15] bought the stability of dusty laminar flow. Datta and Mishra[16] gave the solutions for both the extreme slip conditions. Vajravelu and Nayfeh[17] considered the dusty flow with MHD. Hamid *et al.,*[18] illustrated the second solution for the particle fluid in a boundary layer flow. Mishra and Rauta[19] understood that when the parameter for unsteady dusty fluid increases boundary layer thickness decreases. Gireesha *et al.*,[20] studied the two-point boundary problem for a suspended particle fluid in the presence of MHD and time dependent. An analytical solution was presented for a dusty viscoelastic fluid by varying mass diffusion considering the Forcheimmer model by Uma *et al.,*[21]. Elbashbeshy *et al.*,[22] examined the course of magnetic dusty particles with thickness being varied. Shankaralingappa *et al.*,[23] analysed the steady hyperbolic dusty flow and found that the velocities of particle suspended fluid and fluid diminishes with increase in Forchheimer force. Jalil *et al.*,[24] showed that increasing magnetic effect drags the velocity by an exact solution. Mallikarjuna *et al.*,[25] studied the two phase hybrid nano fluid incorporating the Forchheimer effect. Sharma and Gandhi[26] analyzed the viscous dissipation effects together with Joule heating on a stretching sheet and showed that the Joule effect is responsible for the increasing thermal energy. A comparison between Williamson and Casson nano fluids were analyzed by Eswaramoorthi *et al.*,[27] and reported that the shear stress is more for Williamson nano fluid. Balaji *et al.*,[28] gave the stability of ferromagnetic fluid by considering the system as the function of porous parameter, magnetic field frequency. Thomas *et al.*,[29] studied the temperature modulation effects on ferroconvection when there is a magnetic field.

The scares of papers on the effect of forchheimer effect on two phase flow with fluid properties not constant with MHD made an attempt to analyse the same with double diffusion. The effect has been incorporated graphically for better understanding of the flow. The author has compared the result of the study with Pal and Shivakumara[7] in the absence of dusty parameters and are in good agreement.

2.0 Mathematical Formulation

An incompressible, steady, bi-dimensional flow is considered. The x and y-axes are measured along and perpendicular to the plate respectively. , the free stream velocity is considered in an upward direction and the acceleration due to gravity acts vertically downwards. The concentration temperature near the plate are always more than and at the far away region. The dust particle is freely suspended in the fluid and always the number density of the dusty fluid is presumed to be constant. Given below are the governing equations invoking the boussinesq approximations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\vec{g}\beta_T(T-T_\infty)+\vec{g}\beta_C(C_\infty-C)+\frac{\bar{\mu}}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{\rho}\frac{\varepsilon(y)}{k(y)}(u-U_0)$$
$$+\frac{DK(U_d-u)}{\rho}+\frac{\sigma_m B_0^2\varepsilon(y)(U_o-u)}{\rho}+\frac{c_b\varepsilon^2(y)}{\sqrt{k(y)}}({U_0}^2-u^2) \tag{2}$$

$$u_d\left(\frac{\partial u_d}{\partial x}\right)+v_p\left(\frac{\partial u_d}{\partial y}\right)=\frac{K}{m}(u-u_d) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha(y)\frac{\partial T}{\partial y}\right)+\frac{\bar{\mu}}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(\gamma(y)\frac{\partial C}{\partial y}\right) \quad (5)$$

Corresponding conditions are:

$$u=0, v=0, T=T_W, C=C_W \text{ at } y=0,$$
$$u=U_0, u_d=U_0, v=0, T=T_\infty, C=C_\infty \text{ as } y\to\infty \quad (6)$$

The velocity components along the axis are given by:

$$u=U_o f'(\eta),\; v=\frac{-1}{2}\sqrt{\frac{\upsilon U_o}{x}}[f(\eta)-\eta f'(\eta)],\; u_d=U_o g'(\eta) \quad (7)$$

The following similarity transformations along with (7) are used to solve the equations from (2) to (5)

$$\eta=\left(\frac{y}{x}\right)\left(\frac{U_o x}{v}\right)^{1/2},\; \psi=\sqrt{\upsilon U_o x} f(\eta)\; \phi=\frac{C-C_\infty}{C_W-C_\infty},\; \theta=\frac{T-T_\infty}{T_W-T_\infty} \quad (8)$$

The properties of fluids are considered as

$$\varepsilon(\eta)=\varepsilon_o(1+d^*e^{-\eta});$$
$$\gamma(\eta)=\gamma_o\left[\varepsilon_o(1+d^*e^{-\eta})+\gamma^*\{1-\varepsilon_o(1+d^*e^{-\eta})\}\right];$$
$$k(\eta)=k_o(1+de^{-\eta});$$
$$\alpha(\eta)=\alpha_o\left[\varepsilon_o(1+d^*e^{-\eta})+\sigma^*\{1-\varepsilon_o(1+d^*e^{-\eta})\}\right] \quad (9)$$

The magnitude of d (uniform permeability) and d^* (Variable permeability) are taken as 3.0 and 1.5 as stated by Mohammadein and El-Shaer[2] and Nalinakshi *et al.*,[8]. Using equation (6) and similarity transformations equation (7) in equations (2) to (5), the following equations are obtained.

$$f'''+\frac{1}{2}ff''+Ri(\theta-N\varphi)\frac{\alpha^*(1+d^*e^{-\eta})}{\sigma\mathrm{Re}(1+de^{-\eta})}(1-f')+\frac{\beta^*(1+d^*e^{-\eta})^2}{(1+de^{-\eta})^{1/2}}(1-f'^2)+\alpha(g'-f')+M^2\varepsilon_0(1-f')(1+d^*e^{-\eta})=0 \quad (10)$$

$$\frac{1}{2}gg''+\beta(f'-g')=0 \quad (11)$$

$$\theta''=\frac{-\frac{1}{2}\Pr f\theta'-\varepsilon_o e^{-\eta}d^*(\sigma^*-1)\theta'-\Pr f''^2E}{\varepsilon_o+\sigma^*(1-\varepsilon_o)+\varepsilon_o d^*e^{-\eta}(1-\sigma^*)} \quad (12)$$

$$\phi''=\frac{-\frac{1}{2}Scf\phi'+\varepsilon_o d^*e^{-\eta}\phi'(\gamma^*-1)}{\varepsilon_o+(1-\varepsilon_o)\gamma^*+\varepsilon_o d^*(1-\gamma^*)e^{-\eta}} \quad (13)$$

Where R_i = Gr/Re^2 = Richardson number, $\Pr=\bar{\mu}/\rho\alpha_o$ is prandtl number, $N=\frac{\beta_c(C_w-C_\infty)}{\beta_T(T_w-T_\infty)}$ is Buoyancy ratio, $\mathrm{Re}=U_o x/v$ = Reynolds number, $\frac{KDx^*\upsilon}{\rho U_0^2}=\alpha$ = concentration parameter and the term $\frac{K\upsilon x^*}{mU_0^2}=\beta$ = interaction parameter of the particle. $Sc=\frac{\bar{\mu}}{\rho\gamma_0}$ = Schmidt number. M^2 = magnetic parameter, $E=U_o^2/C_p(T_w-T_\infty)$ = Eckert number.

The boundary conditions which are transformed is given as:

$$f=0, f'=0,\ \theta=1,\ \phi=1, \text{ at } \eta=0$$
$$f'=1,\ g=f, g'=1,\ \theta=0,\ \phi=0, \text{ at } \eta\to\infty \quad (16)$$

3.0 Methodology

As the equations from (10) to (13) are coupled and nonlinear with singular points, they are converted to ordinary differential equations through similarity transformations. With the help of numerical approach known as shooting method the transformed equations are solved.

$$f=f_1$$

$$\frac{\partial f_1}{\partial\eta}=f'=f_2,\ \frac{\partial f_2}{\partial\eta}=f''=f_3$$

$$\theta=f_4,\ \frac{\partial f_4}{\partial\eta}=\theta'=f_5$$

$$\phi=f_6,\ \frac{\partial f_6}{\partial\eta}=\phi'=f_7$$

$$g=g_1=f_8$$

$$\frac{\partial f_8}{\partial\eta}=g'=f_9$$

$$f'''=-1.0*(0.5*f_1*f_3)+Ri(f_4-N*f_6)-\alpha^*/\sigma\mathrm{Re}\frac{(1+d^*e^{-\eta})}{(1+de^{-\eta})}(1-f_2)-\alpha(f_9-f_2)+M^2\varepsilon_0(1+d^*e^{-\eta})(f_2-1)-\frac{\beta^*(1+d^*e^{-\eta})^2}{(1+de^{-\eta})^{1/2}}(1-f_2^2)$$

$$g''=-2\beta\left(\frac{f_2-f_9}{f_8}\right)$$

$$\theta''=\frac{-1/2\Pr f_5f_1-\varepsilon_0d^*e^{-\eta}(\sigma^*-1)f_5-\Pr f_3f_3E}{\varepsilon_0+\sigma^*(1-\varepsilon_0)+\varepsilon_0d^*e^{-\eta}(1-\sigma^*)}$$

$$\phi'' = \frac{-1/2 f_7 f_1 Sc - \varepsilon_0 d^* e^{-\eta}(\gamma^* - 1) f_7}{\varepsilon_0 + \gamma^*(1-\varepsilon_0) + \varepsilon_0 d^* e^{-\eta}(1-\gamma^*)}$$

Boundary Conditions:

$f_1 = 0, f_2 = 0, f_4 = 1, f_6 = 1$, at y = 0

$f_2 = 1, f_4 = 0, f_6 = 0, f_8 \rightarrow f_1, f_9 = 1$ at y → ∞

4.0 Results and Discussion

A double diffusive convection in dusty fluids with MHD is analyzed with Forchheimer effect. The values of various parameters are calculated numerically. The value of Ri = 0.2, α^*/σ Re = 0.4, Pr =0.71, N=1, Sc=0.22 , M^2=0.2, α =0.2 , β=0.2, β^* =0.5, E=0.1 are fixed values considered for all the variations. The code validity is checked by comparing the results obtained with already obtained results with M=0, α =0, β=0, N=0, β^*=0 which is shown in Table 1. The values of various parameters for VP case is shown in Table 2.

4.1 Variation of Gr/Re^2

From Figures 1 and 2 the velocity of both the phases decreases which is because of the Forchheimer applied.

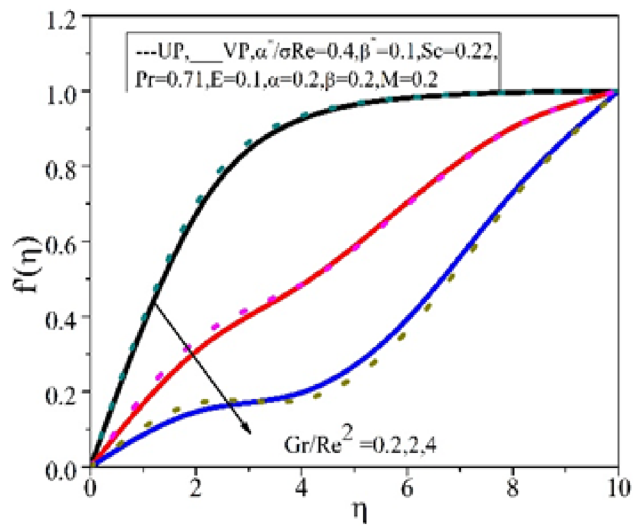

Figure 1. Influence of Gr/Re^2.

Table 1. Comparison of $f''(0)$ and $-\theta'(0)$ for validity of code

σ^*	Gr/Re^2	α^*/σRe	Pal and Shivakumara[7]		Present study	
			$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
2.0	0.2	0.0	0.611321	0.381233	0.61238	0.380067
		0.1	0.667804	0.386040	0.66937	0.38377
		0.5	0.846341	0.417658	0.85389	0.39276
	0.5	0.0	0.958156	0.403083	0.958389	0.402965
		0.1	0.987898	0.406430	0.992109	0.403479
	2.0	0.0	2.415691	0.376339	2.31569	0.403734
4.0	0.2	0.0	0.627031	0.504676	0.629265	0.501355
		0.1	0.681575	0.507192	0.682956	0.504583
		0.5	0.859094	0.519451	0.860844	0.512688
	0.5	0.0	0.993653	0.528572	0.993653	0.528672
		0.1	1.022091	0.528510	1.02209	0.5285099

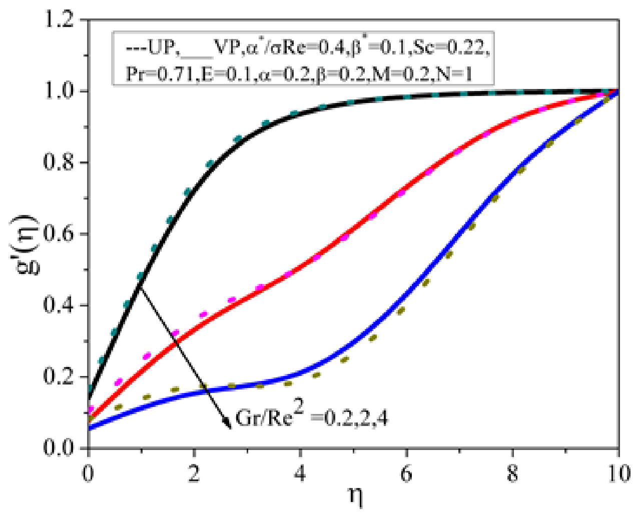

Figure 2. Velocity of dusty fluid based on Ri.

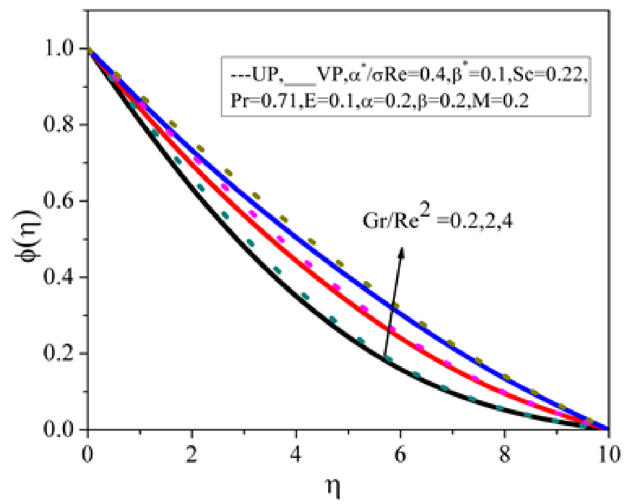

Figure 4. Concentration based on Gr/Re^2.

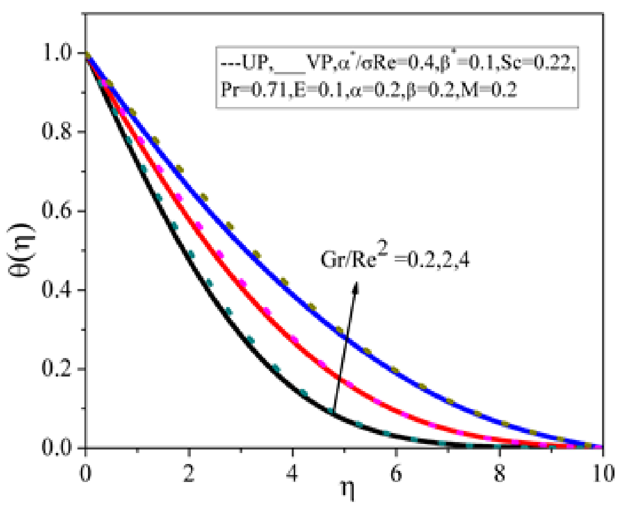

Figure 3. Temperature based on Gr/Re^2.

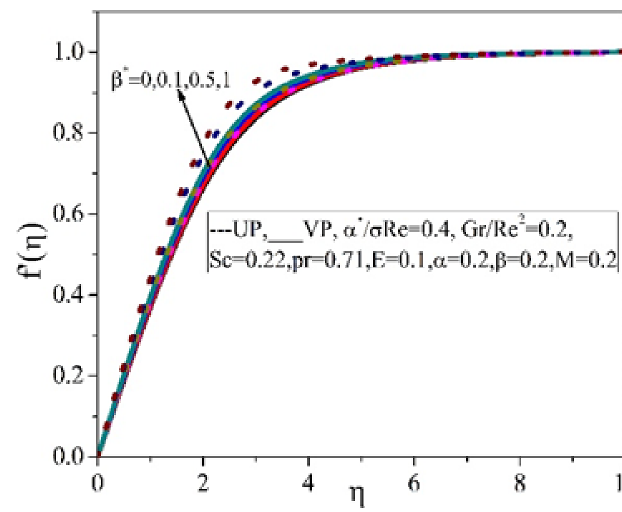

Figure 5. Velocity based on inertial parameter.

It is also found that the velocity for UP case is more than the variable permeability initially but latter the velocity for UP decreases than VP and for lower values of Gr/Re^2, UP and VP coincide with each other. Since the velocity decreased in both cases, as seen in Figures 3 and 4, the temperature and concentration profile increased. The UP case is more dominant in the starting but latter they both coincide.

4.2 Variation of β*

Both the fluid's and the dusty fluid's velocity increase with increasing inertial parameter value, as shown in Figures 5 and 6. This is because of the inertial force in the

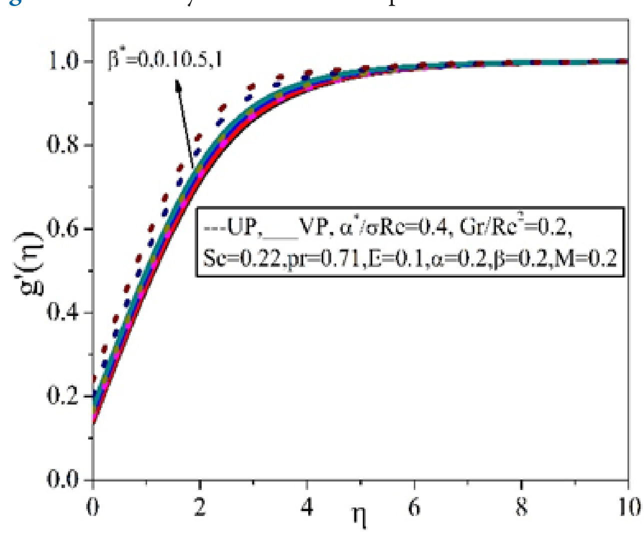

Figure 6. Velocity of particle based on β*

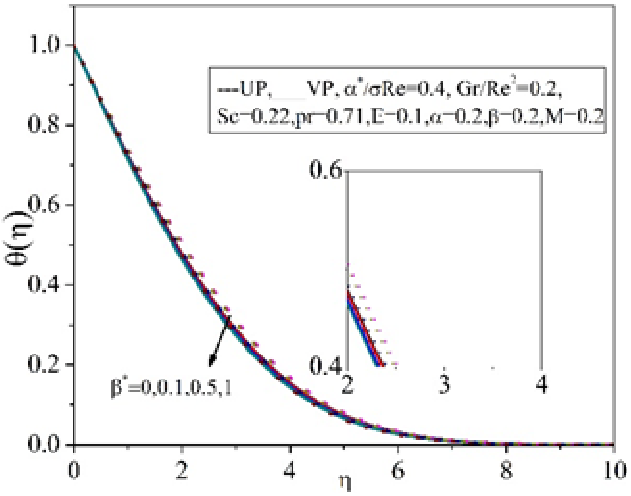

Figure 7. Temperature based on β*

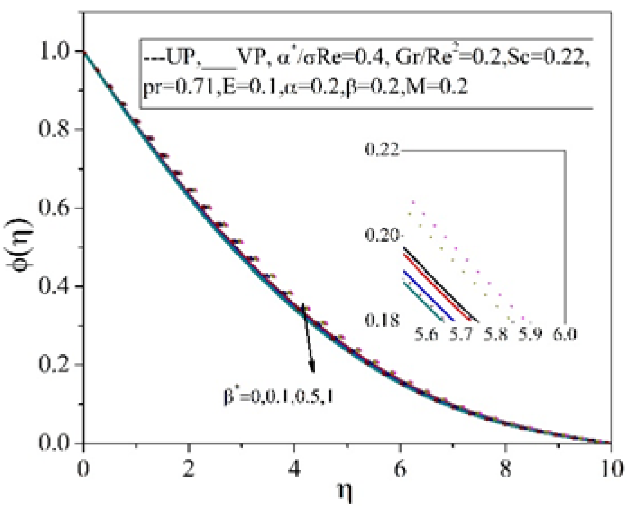

Figure 8. Concentration based on β*

direction of the flow. The temperature and concentration as seen in Figures 7 and 8 decreases with increases in the parameter as the boundary layer thickens. Initially UP case is more than the VP case latter Variable permeability shows greater effect. We could observe that the UP case is more dominant than the VP cases in the concentration profile.

4.3 Variation of Prandtl Number and M

The heat transfer is more as the temperature increases which is evident from Figure 9. Hence even lower value of Prandtl number can transfer more amount of heat. As can be observed from Figures 10 and 11, the velocity

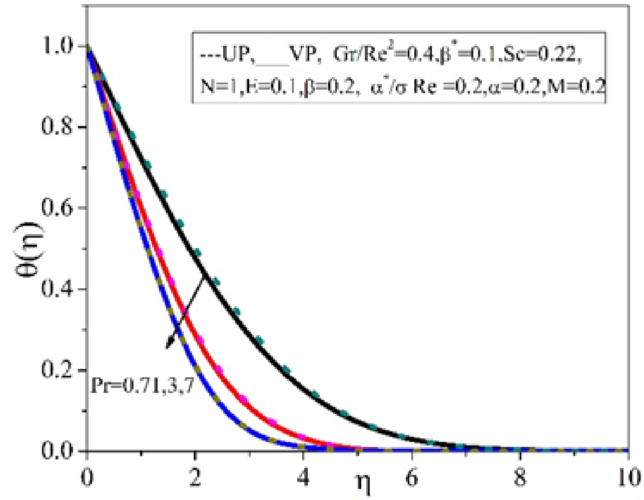

Figure 9. Influence of Prandtl Number.

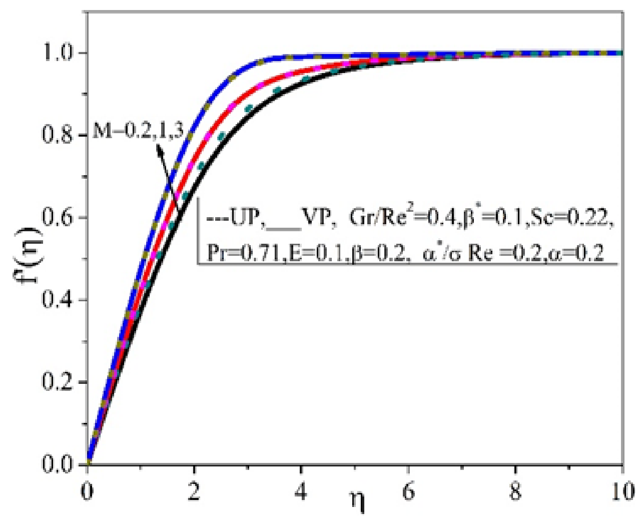

Figure 10. Velocity based on M.

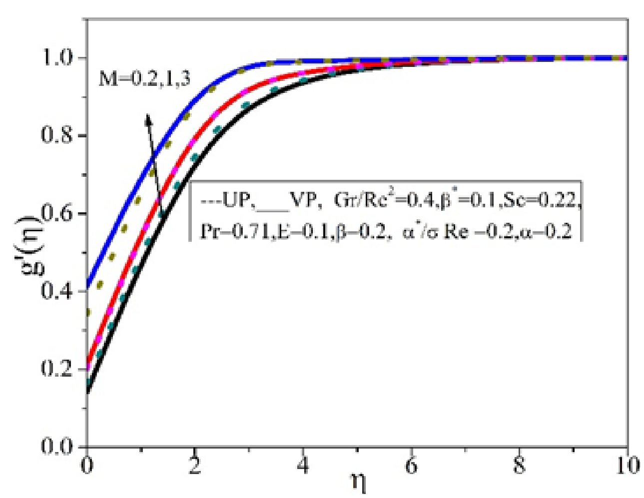

Figure 11. Dusty velocity based on M.

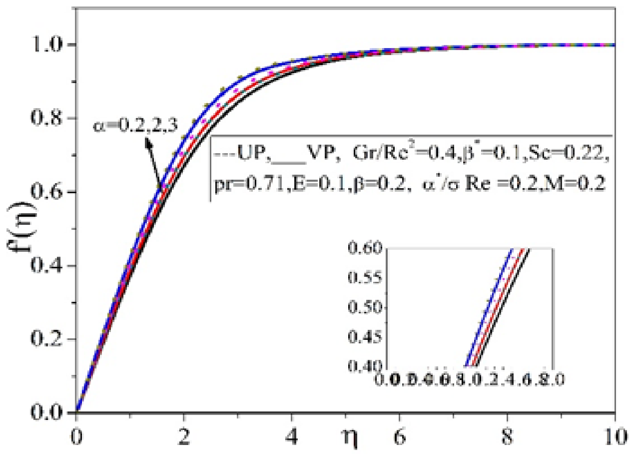

Figure 12. Velocity based on concentration parameter.

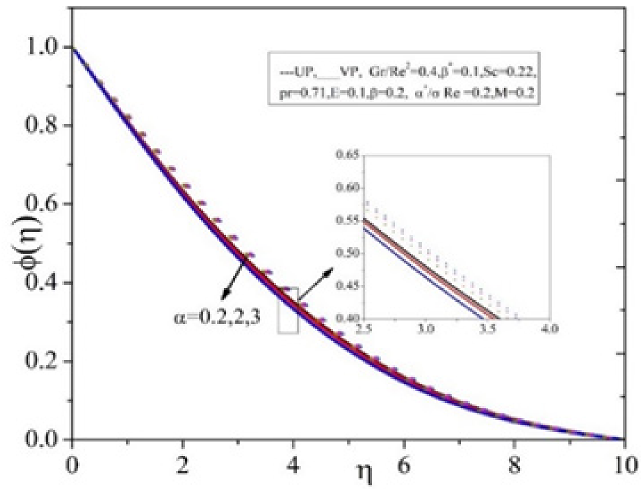

Figure 15. Concentration based on α.

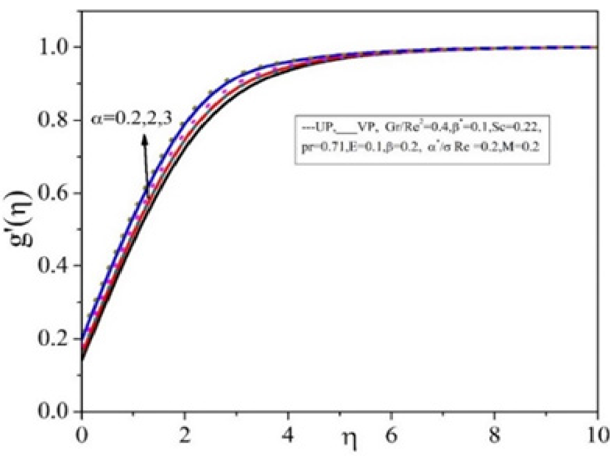

Figure 13. Dusty velocity based on α.

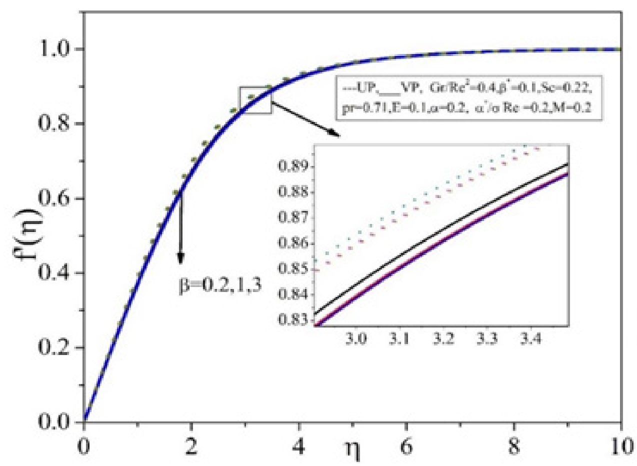

Figure 16. Velocity based on β.

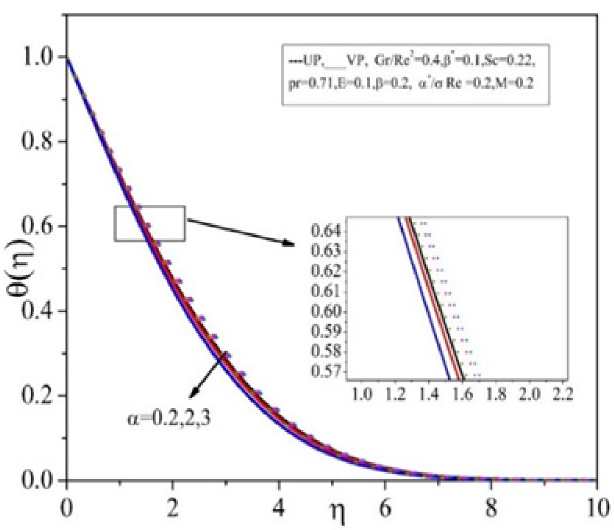

Figure 14. Temperature based on α.

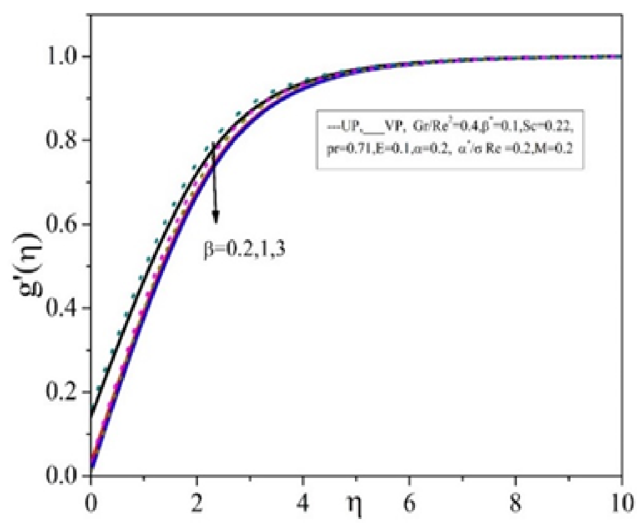

Figure 17. Influence of β.

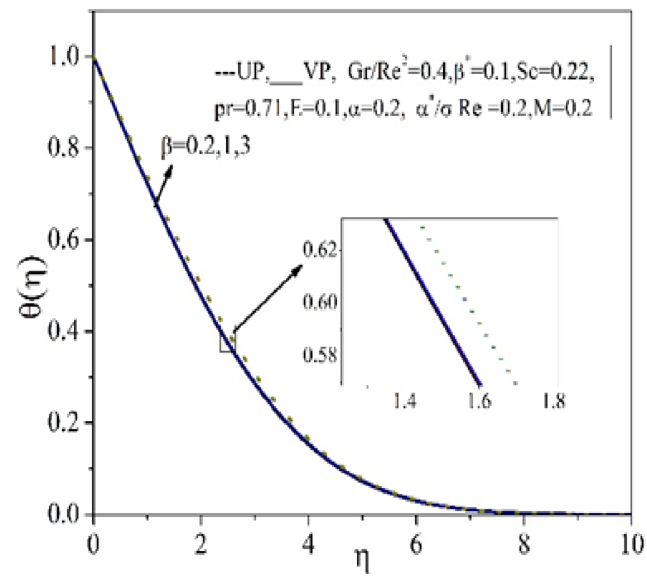

Figure 18. Temperature based on β.

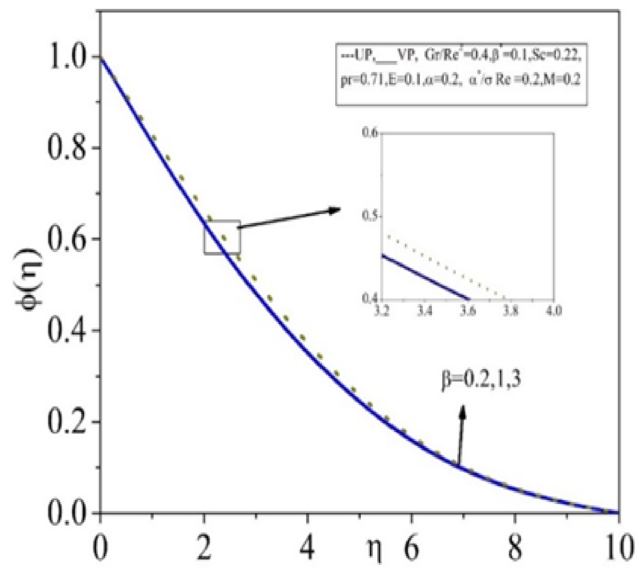

Figure 19. Concentration based on β.

Table 2. Values of various parameters considering Variable Permeability (VP) case

$\alpha^*/\sigma Re$	Gr/Re^2	M	β^*	α	β	f"(0)	-θ' (0)
0.4	0.2	0.2	0.1	0.2	0.2	0.64986	0.37401
	0.6					0.55745	0.35912
	2.0					0.38743	0.29687
	4.0					0.687469	0.38062
	6.0					0.26865	0.21243
0.1	0.2					0.46586	0.35716
2.0						1.21898	0.39752
0.4		1				1.08971	0.39109
		3				2.92974	0.38706
		0.2	0.0			0.62003	0.372141
			0.5			0.747654	0.37882
			0.1	1		0.74288	0.3798
				3		0.98868	0.39009
				0.2	1	0.63237	0.37277
					3	0.62908	0.37253

rises as M increases for both fluid and dusty fluid. This is because of the increased electric currents by increased Lorentz force.

4.4 Variation of Concentration Parameter

As seen in Figures 12 and 13 the velocity of fluid and particles increase with increase in the concentration parameter which is practically seen in sedimentation. The temperature and concentration from Figures 14 and 15 decreases with the higher value of the parameter in both UP and VP cases. This is because as the number of particles increases, each particles receives less heat energy from the fluid.

4.5 Variation of Interaction Parameter

As demonstrated in Figures 16 and 17, the fluid and particle velocities decrease as the fluid interaction parameter increases because it is based on the amount of time it takes for the suspended particles to interact with the fluid. From Figures 18 and 19 the temperature and concentration respectively increases. Here UP dominates the flow and this parameter does not show much effect.

5.0 Conclusions

An analysis of Forchheimer effect with induced magnetic field on a double diffusive mixed convective dusty fluid is made here. With the aid of similarity transformations, the non-linear and coupled governing equations are transformed to ODE which are further solved numerically using a well-known shooting method followed with Runge-Kutta method. The results obtained in this analysis is listed below.

1. The velocity of both phases increases as inertial parameter increases which plays a vital role in hydro carbon resource coal mining, shale gas and underground mining.
2. Due to the particles in the fluid the magnetic parameter increases f'', velocity of the fluid and g', velocity of the particle. The heat energy and concentration decreases with M.
3. The velocity decreases, when the fluid interaction parameter is increased. But the concentration and temperature increases.
4. The growing impact of the particle's concentration parameter enhances the velocity of the fluid and the dusty fluid whereas a reverse effect *i.e.* the decrease in the concentration and temperature of the fluid is seen.

6.0 Acknowledgement

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