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MHD and Cross-Diffusion Fluid Flow for Free Convection of Casson Fluid Due to A Stretching Cylinder

R. N. Shuba[1,2]*, D. V. Jayalakshmamma[2], P. A. Dinesh[3], S. Sushma[4] and T. C. Sushma[5]

[1]*Department of Mathematics, Sir M. Visvesvaraya Institute of Technology, Bengaluru – 560064, Karnataka, India; subha_math@sirmvit.edu*

[2]*Department of Mathematics, Vemana Institute of Technology, Bengaluru – 560034, Karnataka, India; jaya.dvj@gmail.com*

[3,4]*Department of Mathematics, Ramaiah Institute of Technology, Bengaluru – 560054, Karnataka, India; dineshdpa@msrit.edu, sush.msrit@gmail.com*

[5]*Department of Mathematics, Nitte Menakshi Institute of Technology, Bengaluru – 560064, Karnataka, India; sushmadhuadekandi@gmail.com*

Abstract

The present communication is dedicated for understanding the control of Lorentz force, Soret, Dufour effect and body force on the convection-free flow of Casson fluid on heat and mass transfer due to a stretching cylinder. For the considered two-dimensional physical configuration, a non-linear partial differential equation which are coupled are derived as a governing equation to study the behaviour of concentration, temperature and velocity which in turn modified to differential equations which are ordinary and non-linear by employing a suitable similarity transformation. A well-known shooting technique is employed to understand the characteristic variations of the fluid flow with the influence of some of the non-dimensional parameters viz., Schmidt's, Reynolds number, Soret and Dufour parameters, Hartmann number, Prandtl number. These parameters take a vital role in controlling fluid flow, heat and mass transfer in various real problem applications. The present results are contrast with the results of earlier study as a particular case and good agreement is found.

Keywords: *Casson Fluid, Free Convection, Soret and Dufour Effect*

1.0 Introduction

Due to vast applications in engineering, biological fields and industrial like manufacturing of paints, metallic extrusions, lubricants, China clay, pharmaceutical, mines of petroleum, food processing, fields of power generation, fusion research, MHD accelerators and many more has gained significant attention on MHD flows. MHD is to study the flow behavior by disturbing the fluid flow in a certain required direction by varying the formation of layer on boundaries.

For variety of flow models with hydrodynamic flow across a sheet is stretched has procured recognition of many researchers as to its tantalizing uses in engineering and metallurgical area. Because of these applications profound research has been conducted like Wang has investigated research on flow over a tensile cylinder placed in a direction vertically with convection which is

**Author for correspondence*

natural[1]. Ishak *et al.,* has given the numerical solution for Heat transfer and MHD flow over a shrinking cylinder[2]. Abbas *et al.,* has studied the Magnetohydrodynamic flow with radiant heat effects of tensile cylinder within a penetrable medium[3]. Fetecau *et al.,* has worked on general solution of hydromagnetic free transmission flow across a huge plate along with chemical reactions and Newtonian heating[4]. Loganathan *et al.,* has investigated Magnetohydrodynamic effect of the transmission free flow of a partly finite moving cylinder which is placed vertically along with the varying temperature[5]. Dinesh *et al.,* has done research on dissipative squeezing unsteady flow in between plates which are placed parallelly under the influence of thermal radiations[6]. In petroleum mines, metal extrusion, polymer engineering, and certain separation procedures are among the industries that use casson fluid. Eldabe *et al.,* has studied the MHD energy transfer of Casson fluid amid of two cylinders which are rotating[7]. Jayalakshmamma *et al.,* has analyzed the flow fluid which is viscous when magnetic field is applied over an impervious cylinder which is porous in nature[8].

Among the many non-Newtonian nanofluids, Casson is one of the fluids where strain and stress are related non-linearly. Casson fluid model helps in depression of heat that decides the quality of the final products and has many applications in polymer industries in production of glass fibre, extraction and production of polymers, rubber sheets and in printing ink production. The blood that flows in human body, jelly, honey, sauce are few examples of Casson fluids.

Casson was first to introduce the Casson to investigate flow nature of suspension of oil and pigment[9]. In depicting slurry of cement, production of petroleum in many more, the model of Casson fluid caught an attention of many engineers due to its unique way in prediction of higher rate of shear viscosities with availability of moderate or low rate in shear data[10]. Mahdy has studied the energy transfer of flow of a Casson fluid due to extending cylinder with Soret and Dufour effects[11]. Gireesha *et al.,*[12] has discussed the impact of viscous dissipation on MHD flow, heat and mass transfer of Casson fluid over a plate by considering mixed convection. Alam *et al.,*[13] has observed the mass transfer due to steady free convection flow over a pervious plate placed vertically which is porous immersed in a permeable medium influenced by diffusion-thermo and thermo-diffusion effect. Shilpa *et al.,*[14] has discussed the analytical solution of flow with mixed convection of Casson fluid sink in a penetrating medium. Sushma *et al.,*[15-16] discussed the casson fluid flow of stretching sheet with chemical reaction, triple diffusion over a vertical plate. Postelnicu[17] has examined the thermo-diffusion and diffusion-thermo effect on mass and energy transfer of vertical wall by applying magnetic field.

The study on flow nature of Casson fluid primarily with MHD effect with mixed convection flow has grabbed the attention of numerous researchers. Boundary-layer Casson fluid flow over a shrinking/ stretching cylinder is a pertinent category of flow recognized in multitude engineering and industrial applications. It is noticing that in numerous literatures, less analysis is accomplished by inclusion of body force and Lorentz force to model of Casson flow across cylinder when it is stretched. The current paper gives the study of electrically conducting, axisymmetric flow of incompressible non-Newtonian Casson fluid over stretched cylinder with existence of transverse direction of magnetic field is put into flow with additional effect of body force to the basic model[11] to improvise the flow model to investigate better results. Vidya Shree *et al.,*[18] investigated a study on perturbation technique at how electric modulation affected a dielectric fluid that was saturating a pervious media. The system's stability is shown by a corrected Rayleigh number, which is determined by the electric, thermal, and porosity parameters in addition to the frequency of the electric field modulation. To forecast the impact of the spacing distance between two cylinders on the flow-induced behavior researchers like Sudip Chakraborty *et al.,*[19] numerically examined the effect of gap ratio on the flow-induced behavior of two circular cylinders arranged side by side at high Reynolds numbers. This behavior is frequently observed in numerous industrial applications, such as cooling towers, heat exchanger tubes, and cables transmission.

2.0 Mathematical Formulation

For incompressible isotropic flow of Casson fluid, the rheological model

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{P_y}{\sqrt{2\pi}}\right) e_{ij}, \pi > \pi_c \\ 2\left(\mu_B + \dfrac{P_y}{\sqrt{2\pi_c}}\right) e_{ij}, \pi < \pi_c \end{cases}$$

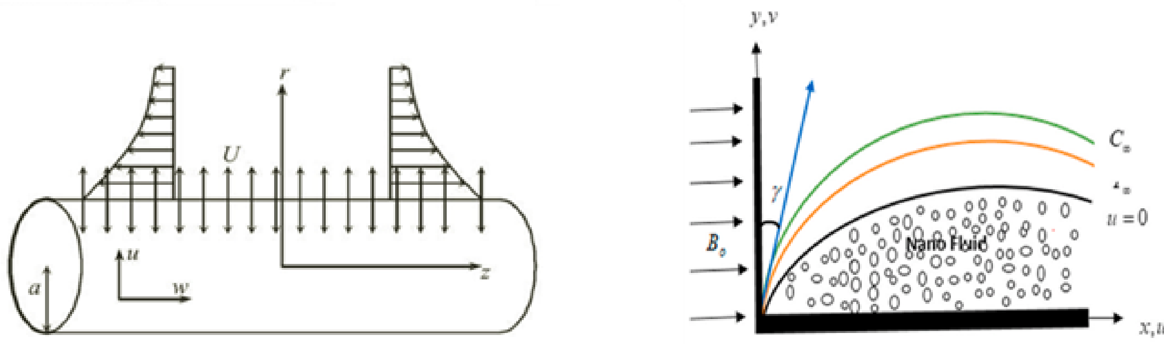

Figure 1. Physical Configuration.

Equation of continuity

$$\frac{\partial(rw)}{\partial z}+\frac{\partial(ru)}{\partial r}=0 \tag{1}$$

Modified Stokes equation

$$w\frac{\partial w}{\partial z}+u\frac{\partial w}{\partial r}=\nu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right) \tag{2}$$

$$w\frac{\partial u}{\partial z}+u\frac{\partial u}{\partial r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\nu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)$$
$$-\vec{g}\rho_0[1-\beta(T-T_\infty)+\ \beta^*(C-C_\infty)]+\mu(\vec{J}\,x\,\vec{B}\,) \tag{3}$$

Energy Equation

$$w\frac{\partial T}{\partial z}+u\frac{\partial T}{\partial r}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+\frac{D_k}{c_p c_s}\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right) \tag{4}$$

Concentration Equation

$$w\frac{\partial C}{\partial z}+u\frac{\partial C}{\partial r}=D\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right)+\frac{D_k}{T_m}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{5}$$

Boundary conditions:

r = a: $u = U = -ba\gamma$, $w = w_w = 2bz$, $T = T_w$, $C = C_w$;

$r\to\infty$; $w\to 0$, $T\to T_\infty$, $C\to C_\infty$

Here b and γ are constants with γ<0 and γ>0 correspond to blowing and suction and respectively.

3.0 Solution for the Problem

By adapting Method of Similarity transformation similar solution to governing equations are found:

$$\eta=\frac{r^2}{a^2},\ w=2bf'(\eta)z,\ u=-\frac{ba}{\sqrt{\eta}}f(\eta) \tag{6}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

Substituting Eq. (6) into Eq. (1), (2), (3) & (4), locally similar O.D.E, s is found

$$\left(1+\frac{1}{\beta}\right)\left(\eta\frac{d^3 f}{d\eta^3}+\frac{d^2 f}{d\eta^2}\right)+Re\left(f\frac{d^2 f}{d\eta^2}-\left(\frac{df}{d\eta}\right)^2\right)=0 \tag{7}$$

$$\left(\frac{1+\beta}{\beta}\right)f''(\eta)-[f(\eta)]^2\frac{Re}{2\eta^2}+\frac{Re}{2\eta}[f'(\eta)f(\eta)]$$
$$-\frac{\nu}{4a^2 b}\frac{Ra_T}{\sqrt{\eta}\,Pr}\theta(\eta)+\frac{\nu}{4a^2 b}\frac{Ra_s}{\sqrt{\eta}\,Pr}\phi(\eta)+\frac{\rho\mu\lambda f'(\eta)\,[Ha]^2}{2}=0 \tag{8}$$

$$\eta\frac{d^2\theta}{d\eta^2}+\frac{d\theta}{d\eta}+\mathrm{Pr}\,Re\,f\frac{d\theta}{d\eta}+PrDf\left(\eta\frac{d^2\phi}{d\eta^2}+\frac{d\phi}{d\eta}\right)=0 \tag{9}$$

$$\eta\frac{d^2\phi}{d\eta^2}+\frac{d\phi}{d\eta}+Sc\,Re\,f\frac{d}{d\eta}+Sc\,Sr\left(\eta\frac{d^2\theta}{d\eta^2}+\frac{d\theta}{d\eta}\right)=0 \tag{10}$$

Here,

Pr - Prandtl number, Re - Reynolds, Sc - Schmidt numbers, Ha - Hartmann number, Ra_T -Thermal Rayleigh number and Ra_S- Solutal Rayleigh number, Sr - Soret and Df - Dufour parameters, λ- Aspect ratio given by $Pr=\nu/\alpha$, $Re=ba^2/2\nu$, $Sc=\nu/D$,

$$Ra_T=\frac{\vec{g}\rho_0\beta(T_w-T_\infty)a^3}{\nu\ \propto},\ Ra_S=\frac{\vec{g}\rho_0\beta(C_w-C_\infty)a^3}{\nu\ \propto},$$

$$[\mathrm{Ha}]^2 = \frac{\sigma {B_0}^2 a^2}{\nu\rho}, Sr = \frac{Dk(T_w - T_\infty)}{T_m \nu (T_w - T_\infty)}$$

$$Df = \frac{Dk(C_w - C_\infty)}{c_p c_s \nu (T_w - T_\infty)}, \lambda = \frac{z}{r}$$

4.0 Discussions

The profile of (velocity) flow rate and energy has been demonstrated graphically from the result of steady axisymmetric flow is electrically conducting and incompressible fluid over extending cylinder with occupancy of constant magnetic field put in the transverse direction of the fluid motion this is due to Lorentz force, incorporated in the equation of momentum is more likely a drag force with the aid of body force to basic model[11]. Method of Similarity transformation is used to denote the field of flow velocity, concentration and energy in flow for Casson fluid.

The graph demonstrates the field of flow meandering at the adaptation stage because interfacial conditions at the boundary helps in layer formation due to slight changes in the values of dimensionless parameters. Figure

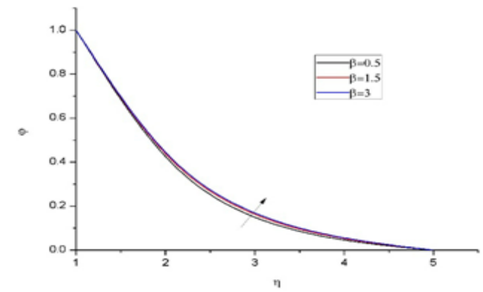

Figure 3. Effect of Casson parameter on Concentration.

1, 2, 3 shows reaction of Casson parameter on flow rate, energy and concentration respectively.

We started slight variations in the β as 0.5, 1.5, 3 then it is observed that velocity rate drops in the flow field. This indicates that the Casson parameter dominates over the rate of flow in suppressing the fluid velocity. That leads to form layers in the velocity profile. As we increase the value of Casson parameter as β=3 velocity boundary layer thickness decreases from η=5 to η=4. This is owing to non-Newtonian fluid which is Casson, that resists fluid motion. As β=0 the non-Newtonian fluid

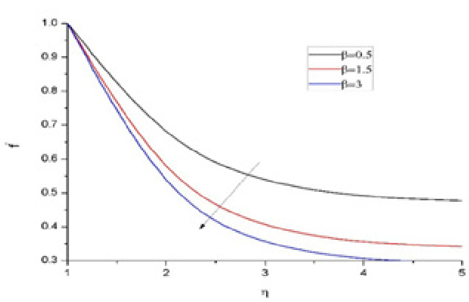

Figure 1. Effect of Casson parameter on velocity.

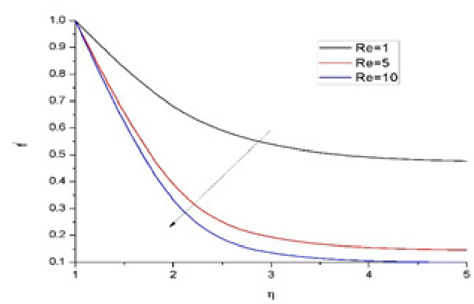

Figure 4. Effect of Reynolds number on Fluid Velocity.

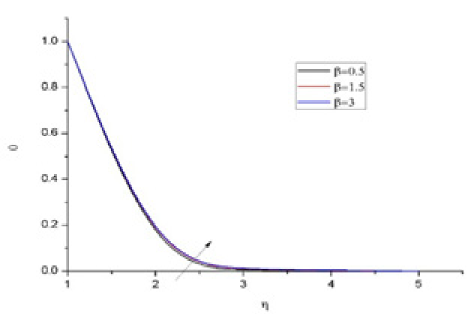

Figure 2. Effect of Casson parameter on Temperature.

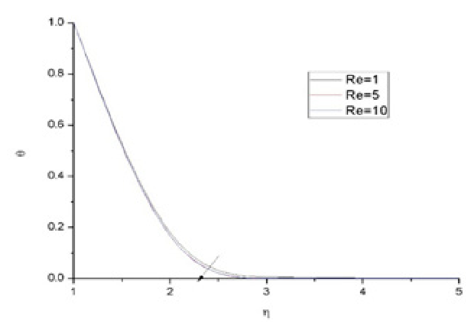

Figure 5. Effect of Reynolds number on Temperature.

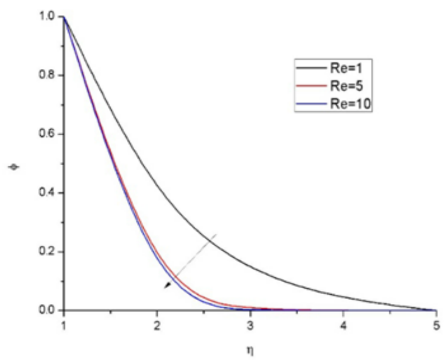

Figure 6. Effect of Reynolds number on Fluid Concentration.

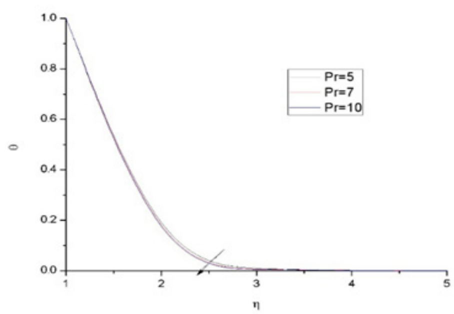

Figure 7. Effect of Prandtl number on Fluid Temperature.

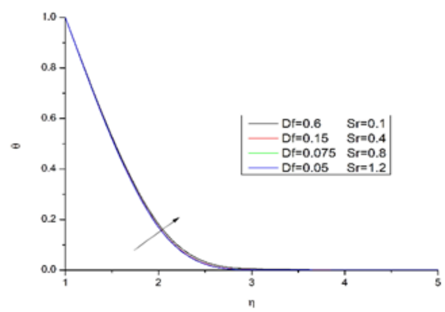

Figure 8. Effect of Soret and Dufour on Fluid Temperature.

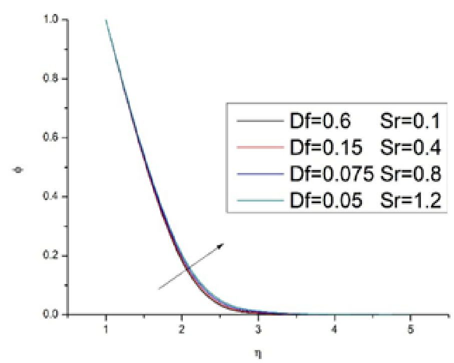

Figure 9. Effect of Soret and Dufour on Fluid Concentration.

behaves as Newtonian. In Figure 2 and Figure 3 depicts the increase in layer thickness at boundary of energy and concentration as values in β increases.

Figures 4-6 exemplifies the control of Reynolds parameter on flow rate (velocity), temperature and concentration. It perceived that intensification in the values of Reynolds parameter decline the thickness of layer at boundary in above listed. This ensues due to rise in Re, the viscosity of the Casson fluid becomes fewer imperative there by velocity distresses fluid flow fields less proliferate. Parameter Re value specifies the corelative importance of inertial effect likened to viscous effect. Thus, thickness of layer at boundary for temperature and concentration also diminutions.

Figure 7 shows the consequence of Pr on Temperature. It signifies, the rise in Pr values from 5 to 10 , the thermal boundary layer width declines from η=3.5 to η=3. This is certitude that Prandtl parameter is proportional to thermal conductivity inversely.

Figure 8 and Figure 9 elucidates the consequence of Dufour and Soret effect on energy and concentration respectively. In Figure 8, as increase in both the effect that diminishes the extent of thermal boundary layer and elevate the extent of concentration boundary layer.

5.0 Conclusions

The influence of MHD on Casson fluid with cross diffusion term motivated the current work is to analyse the behaviour flow through stretched cylinder helps to improvise the flow model for mass and heat transport process holds several fields like oil mines, extrusion of metals, polymer processing engineering, biological, material science and variety uses in many fields like metallurgical process, metal working, glass processing and hot rolling.

6.0 Acknowledgement

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