

Study of Conducting Fluid Flow in Composite Regions Past an Impermeable Sphere in the Presence of Magnetic Field

R. Umadevi¹, D. V. Chandrashekhar², P. A. Dinesh^{3*} and D. V. Jayalakshamma⁴

¹Department of Mathematics, Channabasaveshwara Institute of Technology, Tumkur – 572 216, Karnataka, India

²Department of Mathematics, Vivekananda Institute of Technology, Bengaluru – 560 074, Karnataka, India

³Department of Mathematics, Ramaiah Institute of Technology, Bengaluru – 560 054, Karnataka, India; dineshdpa@msrit.edu

⁴Department of Mathematics, Vemana Institute of Technology, Bengaluru – 560 034, Karnataka, India

Abstract

A steady, two dimensional, incompressible, viscous and conducting fluid flow over a fixed rigid sphere has been considered under the effect of magnetic force applied normal to flow direction. The fluid flow occurs in three multiple regions namely fluid, porous and fluid region respectively. The governing equations are reduced into linear PDEs in terms of dimensionless parameters which intern converted into linear ODEs by similarity transformation method. The impact of Hartmann number and porosity on the fluid flow has been analyzed graphically. It is observed that as the Hartmann number increases for fixed porosity, the flow of fluid is well controlled in porous and non-porous regions. Further, as porosity increases for fixed Hartmann number, fluid flow over a porous region is observed. Also, diminishes the fluid velocity in the porous region due to the suppression of the fluid flow as ' σ ' increases when magnetic field is fixed to finite constant. The same observation is made when the Hartmann number is intensified for the fixed porosity ' σ '.

Keywords: Brinkman Equation, Porosity, Stokes Equation

1.0 Introduction

The fluid flow around/through a porous medium has a significant applications like geothermal engineering, bioremediation, construction engineering, as many more. The two major existing areas of utilizing the porous sources are conversion and conservation of energy as the porous particles are essential for fuels and batteries. Fuel is a substance that can react with other substances to release energy in the form of heat or to do work. This concept was first applied to materials that could release chemical energy, but was later applied to other sources of

thermal energy such as nuclear energy. Lubrication theory is an important application that describes flow of liquids or gases where one dimension is noticeably smaller than others. Lubricants are used as oil in vehicles and kitchens. Lubrication is the use of lubricants to reduce friction and/or contact between two surfaces. Lubrication research is a discipline within the field of tribology. Lubricants can be solids, solid/liquid dispersions, liquids, liquid-liquid dispersions or gases. An important application is the lubrication of mechanical components such as fluid bearings and mechanical seals. Coatings are another

*Author for correspondence

important part of the application and include the preparation of thin films, printings, paints and adhesives.

Solar winds, Dynamo theory and most importantly the well-known application which is MRI scan are some of the greatest important and common areas of study in the MHD field. Many practical problems require a mechanism that use magnetohydrodynamic effects to control the movement of fluids around solid surface. Therefore, Hartmann Flow is a classic problem with important applications in the design of generators and pumps, polymer technology, petroleum refining, also in various heat exchangers. The influence of the magnetic field on the control of flow processes in different regions under different boundary conditions is more familiar. Much attention is paid to magnetohydrodynamic study of flow of conducting fluids in electric and magnetic fields in recent metalworking systems. Hence there is a significant interest to study boundary layer flows in presence of applied magnetic field.

A steady flow of incompressible electrically conducting fluid between two parallel porous disks with magnetic strength applied normal to the disc plane was studied. The solution was obtained using a perturbation method that valid for i) High injection Reynolds number and large Hartmann number¹ ii) high suction Reynolds number and arbitrary Hartmann number² iii) a small suction or injection Reynolds number and an arbitrary Hartmann number³. In all these cases, it was noticed that the external limitation of magnetic field suppresses the movement of the fluid. Later extended their study to MHD flow between a rotating and stationary disk and obtained solutions using perturbation method valid for small Reynolds number. They have shown that azimuthal component of the velocity was strongly influenced by the magnetic field compared to the other two components. In particular, the average normal force on a stationary disc and torque on a rotating disk increase with increasing Hartmann number. The opposite behavior of torque is observed on the stationary disk⁴. The MHD flow of electrically conducting, vertically stratified fluid, over a nonconducting sphere was considered. Drag was calculated for the magnetic and stratification parameters and reveals that the drag obtained by the sphere was increased by increasing the magnetic and stratification parameters⁵. The study of MHD effect on electrically

conducting fluid flow over a rigid sphere surrounded by porous media was carried out. A stream function was used to obtain the analytical solution. They found that increasing Hartmann number to decrease the fluid velocity⁶.

The viscous flow of electrically conducting fluid over a sphere was studied by considering ambient flow fields such as colinear and ambient uniform magnetic field⁷ and drawn the conclusion that the difference between the flow rate of the surrounding particles is greater than the velocity in the Alfvén region and that of the particle velocity below the Alfvén velocity. MHD effect on flow of 2-D, electrically conducting fluid over a rigid sphere surrounded by porous sphere has been analyzed using Stokes and Brinkman's equations⁸. An exact solution was gained by solving the governing equations using similarity transformation method. The control of fluid flow is observed in both porous and nonporous regions by applying magnetic field. The motion of permeable sphere in a spherical vessel in occurrence of uniform magnetic force was studied⁹. Within the porous sphere Brinkman equation and outside the Stokes equation was used. An explicit expressions of stream functions in presence of uniform magnetic field were obtained from both internal and external flow fields. They noticed that, increase in inner porous particle size reduces the drag force. Also, increase in magnetic field decreases the drag coefficient. Further, for high magnetic field, increase in porosity parameter intensify the wall factor.

Full Magnetohydrodynamic (FMHD) steady flow around the circular cylinder was investigated using finite difference method¹⁰. The magnetic field was applied for whole system, with the matching interface conditions. It was noted that the magnetic lines bend inward with an increase in the Reynolds magnetic field, while unbends corresponding to the increase in the interaction parameter. Vorticity contour is reduced by increasing interaction parameter or kinematic Reynolds number. A fully developed flow in a porous system under the uniform Lorentz energy action were performed¹¹. Lorentz power varies vertically due to low fluid fluctuations and special arrangement of magnetic and electric fields on the low plate. The viscous and incompressible fluid flow for dissimilar viscosities was carried out¹². The study of Kelvin-Helmholtz's

linear unsteady of the cylinder-shaped boundary was performed in the presence of saturated magnetic bed structure used in horizontal direction by use of Stokes flow model. It was noticed that although the impact of porous sources on system is not strong, the horizontal electric field has a balancing effect on the system. MHD effect on fully developed and incompressible viscous electrically conducting fluid flow in a porous medium was carried out¹³. Numerical expressions using Galerkin's method for flow rate and volumetric flow in two cases- the Poiseuille and Couette flow were obtained. Impact of various parameters such as Hartmann number, porosity on the velocity profile were discussed. The flow behavior and the effect of magnetic field on porosity of the porous annulus, the conductivity of the inner and outer cylinder was investigated¹⁴. They found that as the Hartmann number increases, velocity profile and induced magnetic field lines were effectively influenced by the conducted cylinders.

Numerical study of viscous incompressible MHD flow of fluid and individual temperature on a flexible flat cylinder mounted in a porous area with generation of internal heat and absorption was studied¹⁵ and analyzed the effects of physical parameters such as velocity, temperature distribution, skin coefficient and Nusselt number on the considered flow. The results obtained shown that increases the velocity profiles of the fluid as the bending parameter increases and decreases as the porosity and the magnetic parameter intensifies. The temperature of the fluid increases by increasing the porous parameter and the magnetic parameter and decreases by increasing the Prandtl number. Also increase in magnetic field results in increasing the skin friction coefficient and Nusselt number. The MHD effect on horizontal, viscous and incompressible fluid flow through a cylinder composed of sparsely filled non-Darcy porous medium on heat convection process including the Joule thermal effect generated by magnetic field was studied using Quasi-numerical method¹⁶. This analysis brings out some conclusions- shear stress at the solid cylinder surface decreases for increasing the Hartmann number and the velocity gradient in the circular cylinder is effectively controlled by the Forchheimer number. The impact of Hartmann number on the flow of a two-dimensional, steady, magneto hydrodynamic fluid in a

circular cylinder enclosed by porous region was studied¹⁷. It was observed that velocity profiles were descending in a porous and non-porous section as the magnetic field increased.

The study dealt with fully developed viscous and incompressible fluid flow in a composite channel which in turn separated equally into two regions was carried out¹⁸. The below region was occupied with flexible porous layer and the above region was clear. They assumed that porosity increases quadratically with the width of the porous layer and explored two significant and actual suitable methods; (i) Poiseuille Flow and (ii) Couette – Poiseuille Flow on a composite region. Exact solutions were obtained for fluid velocity and skin collision. The analysis of influence of Hartmann number on incompressible viscous fluid flow through and over porous sphere implanted in porous area for lower Reynolds numbers was carried out¹⁹. The impact of magnetic field on drag, shear stress and stream lines were demonstrated for different parameters and was discussed through the graphs. Recently the flow of viscous and conducting fluid over a slightly deformed sphere by applying slip boundary conditions at the surface of the sphere was examined and analyzed the effect of some dimensionless parameters²⁰.

The work discussed in the above section are related to study of impact of magnetic field and the porosity on a fluid flow past and through a permeable or impermeable sphere and cylinder by considering single and double regions. With these literatures an attempt is made to study the effect of magnetic field on MHD fluid flow past a solid sphere by considering three regions.

2.0 Mathematical Formulation

The considered flow is 2-D, steady, incompressible, viscous, conducting fluid over a fixed rigid sphere. The rigid sphere of radius a_1 surrounded by a fluid section of radius a_2 bounded by porous sphere of radius a_3 and the whole model is placed in the infinite fluid region is subjected to the transverse magnetic field. Most of the fluids used in industrial applications have very small fluid conductivity and Reynolds magnetic number. Hence the assumption made here is that the induced magnetic field is small compare to applied magnetic field. Also, the

flow is assumed to be axially symmetric. For the above assumptions, the constitutive equations which represents the flow in fluid regions (R_1 & R_3) for $i = 1, 3$ can be written as,

$$\text{Continuity equation: } \nabla \cdot \vec{q}_i = 0 \quad (1)$$

Modified Stokes equation:

$$\nabla p_i = \mu \nabla^2 \vec{q}_i + \mu_h^2 \sigma_e (\vec{q}_i \times \vec{H}) \times \vec{H} \quad (2)$$

In porous region (region R_2) the flow is characterized by:

$$\text{Continuity equation: } \nabla \cdot \vec{q}_2 = 0 \quad (3)$$

Modified Brinkman equation:

$$\nabla p_2 = \bar{\mu} \nabla^2 \vec{q}_2 - \frac{\mu}{K} \vec{q}_2 + \mu_h^2 \sigma_e (\vec{q}_2 \times \vec{H}) \times \vec{H} \quad (4)$$

In addition, we assumed that, $\bar{\mu} = \mu$

We considered the spherical polar co-ordinates (r, θ, ϕ) with the origin as a center of the sphere. All quantities are independent of ϕ since the flow is axially symmetric.

We introduce the following non-dimensional parameters to get dimensionless governing equations:

$$r^* = \frac{r}{a}, \quad \vec{q}_i^* = \frac{\vec{q}_i}{u_\infty}, \quad p_i^* = \frac{a_i p_i}{\mu u_\infty}, \quad \vec{H}^* = \frac{\vec{H}}{H_0} \quad (5)$$

In the view of equation of continuity, a stream function $\psi_i(r, \theta)$ (for $i=1,2,3$ for porous and non-porous regions) is defined as:

$$u_i = \frac{1}{r^2 \sin \theta} \frac{\partial \psi_i}{\partial \theta}; v_i = \frac{-1}{r \sin \theta} \frac{\partial \psi_i}{\partial r} \quad (6)$$

Removing the pressure term from the equations (2) and (4) by cross multiplication method, an equation for ψ_i which is linear partial differential equation of 4th order can be get in the form

$$E^4 \psi_i - J_i^2 E^2 \psi_i = 0, \quad (7)$$

$$\text{where } J_i^2 = \begin{cases} M^2, & i = 1 \\ S^2, & i = 2 \text{ for fluid, porous and} \\ M^2, & i = 3 \end{cases} \quad (8)$$

fluid regions respectively.

The boundary conditions - no-slip condition at the solid spherical core and interfacial conditions at the

boundary of the regions R_1 & R_2 and regions R_2 & R_3 are considered.

No-slip condition at the surface of the rigid sphere is given by:

$$u_3(a_1, \theta) = 0, \quad 0 \leq \theta \leq 2\pi \quad (9)$$

$$v_3(a_1, \theta) = 0, \quad 0 \leq \theta \leq 2\pi \quad (10)$$

and at the interface of region R_1 and R_2 , the boundary conditions are:

$$u_2(a_3, \theta) = u_1(a_3, \theta), \quad 0 \leq \theta \leq 2\pi \quad (11)$$

$$v_2(a_3, \theta) = v_1(a_3, \theta), \quad 0 \leq \theta \leq 2\pi \quad (12)$$

$$\tau_{r\theta(2)}(a_3, \theta) = \tau_{r\theta(1)}(a_3, \theta), \quad 0 \leq \theta \leq 2\pi \quad (13)$$

$$\tau_{rr(2)}(a_3, \theta) = \tau_{rr(1)}(a_3, \theta), \quad 0 \leq \theta \leq 2\pi \quad (14)$$

Also, at the interface of region R_2 and R_3 ,

$$u_3(a_2, \theta) = u_2(a_2, \theta), \quad 0 \leq \theta \leq 2\pi \quad (15)$$

$$v_3(a_2, \theta) = v_2(a_2, \theta), \quad 0 \leq \theta \leq 2\pi \quad (16)$$

$$\tau_{r\theta(3)}(a_2, \theta) = \tau_{r\theta(2)}(a_2, \theta), \quad 0 \leq \theta \leq 2\pi \quad (17)$$

$$\tau_{rr(3)}(a_2, \theta) = \tau_{rr(2)}(a_2, \theta), \quad 0 \leq \theta \leq 2\pi \quad (18)$$

Since the fluid viscosity is equal to Brinkman viscosity, from the equations (14) and (18) we have

$$p_2(a_3, \theta) = p_1(a_3, \theta), \quad 0 \leq \theta \leq 2\pi \quad (19)$$

$$p_3(a_2, \theta) = p_2(a_2, \theta), \quad 0 \leq \theta \leq 2\pi \quad (20)$$

Further, the stream function for uniform flow far away from the boundary is:

$$\psi_1(r, \theta) \sim \frac{r^2}{2} \sin^2 \theta, \quad r \rightarrow \infty \quad (21)$$

3.0 Method of Solution

An equation (21) suggests the similarity solution to the equation (7) as:

$$\psi_i(r, \theta) = f_i(r) \sin^2 \theta, \quad \text{for } i=1,2,3 \quad (22)$$

Solving the equation (7) using an equation (22), to obtain ordinary differential equation of 4th order as,

$$f_i^{iv} - \frac{4}{r^2} f_i'' + \frac{8}{r^3} f_i' - \frac{8}{r^4} f_i - J_i^2 \left(f_i'' - \frac{2}{r^2} f_i \right) = 0 \quad (23)$$

(for J_i^2 , refer the equation (8)).

The corresponding no-slip and interfacial boundary conditions in terms of are given as:

$$f_3(a_1) = 0, \quad (24)$$

$$f_3'(a_1) = 0 \tag{25}$$

$$f_2(a_3) = f_1(a_3) \tag{26}$$

$$f_2'(a_3) = f_1'(a_3) \tag{27}$$

$$f_2''(a_3) = f_1''(a_3) \tag{28}$$

$$f_2'''(a_3) - \sigma^2 f_2'(a_3) = f_1'''(a_3) \tag{29}$$

$$f_3(a_2) = f_2(a_2) \tag{30}$$

$$f_3'(a_2) = f_2'(a_2) \tag{31}$$

$$f_3''(a_2) = f_2''(a_2) \tag{32}$$

$$f_2'''(a_2) - \sigma^2 f_2'(a_2) = f_3'''(a_2) \tag{33}$$

Also, equation (21) becomes,

$$f_1(r) \sim \frac{r^2}{2} \text{ as } r \rightarrow \infty \tag{34}$$

Let the substitution, $g_i(r) = f_i''(r) - \frac{2}{r^2} f_i(r)$ (35)

to reduce ordinary differential equation of 4th order into 2nd order. Here suffix i takes the values from 1 to 3.

Substituting the equation (35) in equation (23), we get,

$$g_i''(r) - \left[J_i^2 + \frac{2}{r^2} \right] g_i(r) = 0 \tag{36}$$

(for J_i^2 , refer the equation (8)).

Further consider the transformation function for $g_i(r)$

as,

$$g_i(r) = \sqrt{r} w_i(r) \tag{37}$$

where the arbitrary function

Using the equation (37) in equation (36), we get

$$r^2 w_i''(r) + r w_i'(r) - \left[\left(\frac{3}{2} \right)^2 + (J_i r)^2 \right] w_i(r) = 0 \tag{38}$$

The solution of equation (38) is obtained as,

$$w_i(r) = C_i I_{3/2}(J_i r) + D_i K_{3/2}(J_i r) \tag{39}$$

where C_i and D_i are arbitrary constants.

Hence the equation (37) becomes,

$$g_i(r) = \sqrt{r} C_i I_{3/2}(J_i r) + \sqrt{r} D_i K_{3/2}(J_i r) \tag{40}$$

From the equations (35) and (40), we get

$$f_i''(r) - \frac{2}{r^2} f_i(r) = \sqrt{r} C_i I_{3/2}(J_i r) + \sqrt{r} D_i K_{3/2}(J_i r) \tag{41}$$

The corresponding gained general solution is given as,

$$f_i(r) = \frac{A_i}{r} + B_i r^2 + C_i \sqrt{r} I_{3/2}(J_i r) + D_i \sqrt{r} K_{3/2}(J_i r) \tag{42}$$

From the equation (42) for $i = 1$, flow in fluid region (region R_1) is given by:

$$f_1(r) = \frac{A_1}{r} + B_1 r^2 + C_1 \sqrt{r} I_{3/2}(Mr) + D_1 \sqrt{r} K_{3/2}(Mr) \tag{43}$$

$a_3 \leq r < \infty$

and the for $i = 2$, flow in permeable region (region R_2) is described by:

$$f_2(r) = \frac{A_2}{r} + B_2 r^2 + C_2 \sqrt{r} I_{3/2}(sr) + D_2 \sqrt{r} K_{3/2}(sr) \tag{44}$$

$a_2 \leq r < a_3$

also, for $i = 3$, flow in fluid region (region R_3) takes the form:

$$f_3(r) = \frac{A_3}{r} + B_3 r^2 + C_3 \sqrt{r} I_{3/2}(Mr) + D_3 \sqrt{r} K_{3/2}(Mr) \tag{45}$$

$a_1 \leq r < a_2$

In fluid region (region R_1) for the fluid flow as $r \rightarrow \infty$ the equation (43) is valid only if $C_1 = 0$. Also, from the equation (34) we get $B_1 = 1/2$. Henceforth equation (43) becomes:

$$f_1(r) = \frac{A_1}{r} + \frac{r^2}{2} + D_1 \sqrt{r} K_{3/2}(Mr), a_3 \leq r < \infty \tag{46}$$

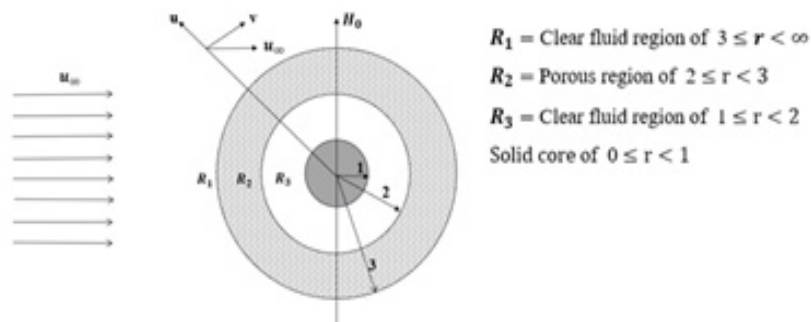


Figure 1. Physical Configuration.

An equations (44), (45) and (46) are involved with arbitrary constants which can be found with help of boundary conditions as given in the equations from (24) to (33).

4.0 Results and Discussions

For the present problem, we analysed the impact of porosity (σ) and Hartmann number (M) on the considered flow

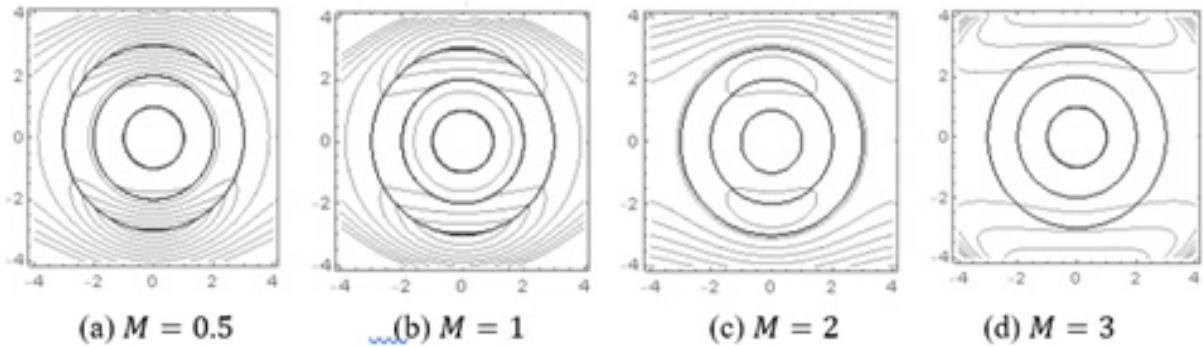


Figure 2. Streamlines for different values of M with $\sigma=1$.

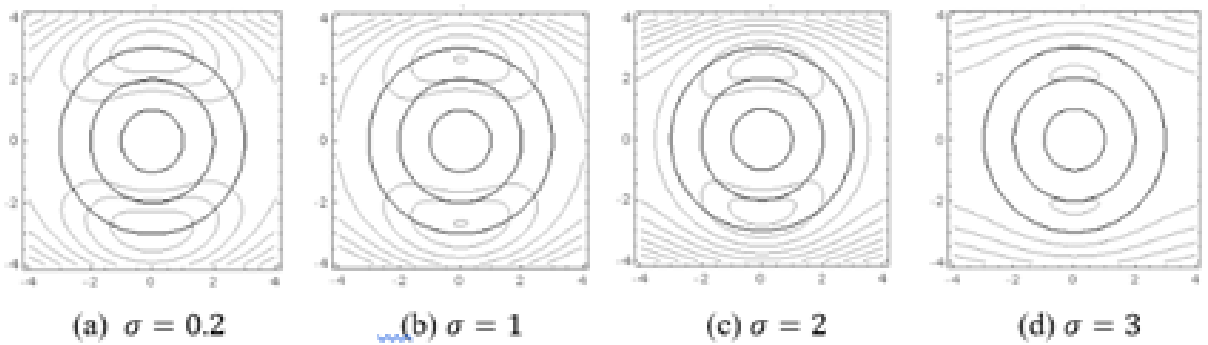


Figure 3. Streamlines for different values of σ with $M=0.1$.

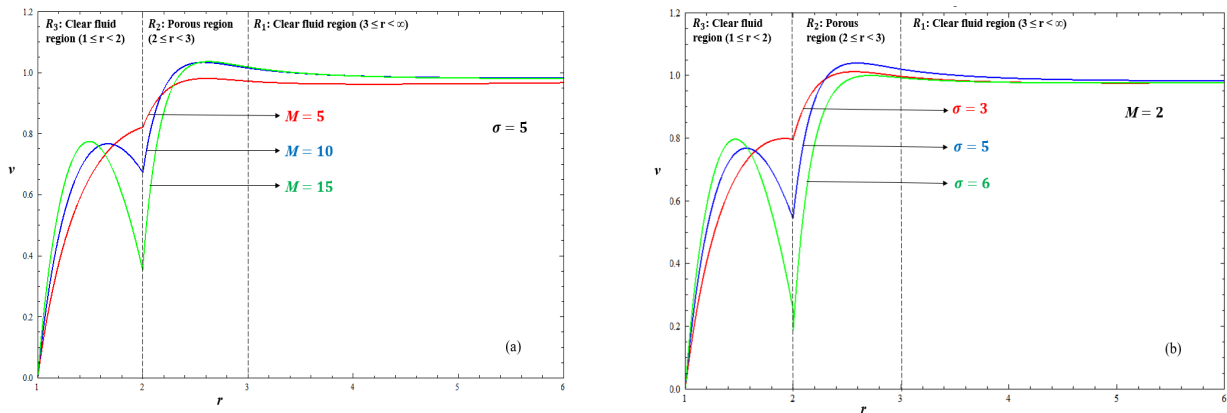


Figure 4. (a). Tangential velocity variations for different values of Hartmann number by fixing porosity. (b). Tangential velocity variations for different values of porosity by fixing Hartmann number

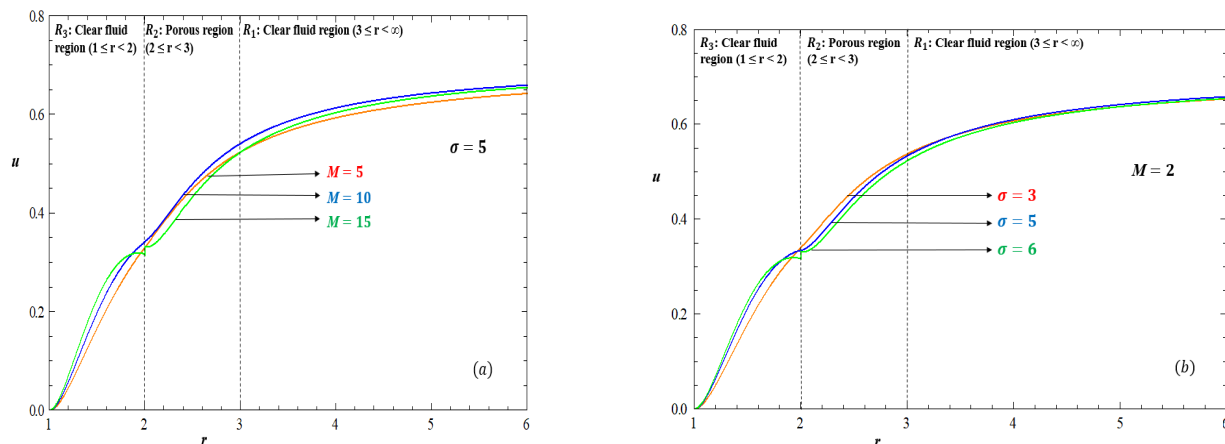


Figure 5. (a). Normal velocity variations for different values of Hartmann number by fixing porosity. (b). Normal velocity variations for different values of porosity by fixing Hartmann number

under the assumptions made and associated boundary conditions which have been mentioned above.

The influence of Hartmann number M and porosity σ on the fluid flow is studied by means of streamlines. First, the influence Hartmann number is studied by fixing the value of σ . For $\sigma=1$ and $M=0.5, 1$ flow of less amount of fluid into porous region is noticed because of low permeability. Thus, the circulatory movement of fluid in the porous region can be seen as shown in Figures 2 (a) and 2 (b). Again, if Hartmann number is increased to $M=2$ (Figure 2 (c)), it was observed that fluid flow in the porous area is still decreased as a result, streamlines are past a porous region. Further Hartmann number is raised to $M=3$ the streamlines are appearing past a solid core. Because increase in Hartmann number suppresses the fluid flow and the same is shown in Figure 2 (d).

The flow behavior is analysed for various values of the porous parameter σ for constant value of Hartmann number M . For negligible Hartmann number the flow behavior is like creepy flow. Further, as porosity is increased, fluid experiences a opposing force to flow in the porous section due to low permeability of the porous medium. Hence the streamlines are away from the solid core. (Figures 3 (a) to 3 (d)).

The tangential velocity of the fluid at boundaries (fluid-porous-fluid) for different values of porosity are analyzed along the line $\theta=\pi/2$. Fluid with uniform velocity far away from boundary flows towards the region of interest. For fixed porosity $\sigma=5$, as the Hartmann number is

amplified velocity diminishes gradually from the uniform speed and increases in clear fluid region (region R3). In this region velocity profile appear in the form of parabola which meets the applied boundary conditions (Figure 4 (a)). The same observation is done for the varying the porosity σ by fixing the value of magnetic field $M=2$. The impact of porosity diminishes the fluid velocity in porous region which is depicted in the Figure 4 (b).

The variation in normal velocity component is studied along the line $\theta=\pi/4$ in Figures 5 (a) and 5 (b). As the Hartmann number increases for fixed porous parameter the normal velocity decreases in porous region from the uniform speed and increases at the interface and again decreases in the fluid region. Also, the variation of normal velocity component is studied for different porous parameter for constant value of Hartmann number in all the three regions. In the porous region normal velocity declines from the uniform speed and rises at the interface and decreases in fluid region.

5.0 Conclusions

The analysis of magnetohydrodynamic flow finds application in MHD power generators, soil science, chemical engineering, planetary magnetospheres and stellar, plasma flow of aeronautical and electronics. Coal-fired MHD systems is one of the major types of MHD power generators. Coal MHD systems uses the coal as fuel to produce a plasma in which the

coal is burned at a temperature sufficient for thermal ionization.

In consideration of mentioned applications, an analytical solution to an incompressible viscous conductive fluid flow around a rigid sphere bounded by a fluid medium surrounded by a porous medium and the entire model is placed in infinite fluid region is presented. The influence of the dimensionless parameters like porosity and Hartmann number on the various flow characteristics of the considered fluid is analyzed. The following conclusions are obtained:

1. Increase in Hartmann number for fixed porosity, declines the fluid flow in the porous and non-porous regions. As a result, streamlines past a solid core is noticed. Correspondingly fluid velocity components also decreased.
2. As the porosity increases for fixed Hartmann number, permeability of porous region is decreased. Hence fluid flow in porous section is reduced. Thus, streamlines past a porous region is noticed and correspondingly fluid velocity components also declines in the porous region.

6.0 Acknowledgement

The author would like to thank Vivekananda Institute of Technology and Channabasaveshwara Institute of Technology for their support to carry out the research work smoothly.

7.0 References

1. Chandrashekara BC, Rudraiah N. Magnetohydrodynamics laminar flow between porous disks for large injection. Reynolds number - Bulletin of Accd. Sci. Georgian SSR. 1969; 53(2):286-9.
2. Rudraiah N, Chandrashekara BC. Flow of conducting fluid between porous disks for large suction. Reynolds number - J. Phys. SOC, Japan. 1969; 27(4):1041-5. <https://doi.org/10.1143/JPSJ.27.1041>
3. Rudraiah N, Chandrashekara BC. MHD laminar flow between porous disks. Applied Sci. Res. 1970; 23:42-52. <https://doi.org/10.1007/BF00413186>
4. Chandrashekara BC, Rudraiah N. Three-dimensional magnetohydrodynamic flow between a rotating and stationary disk with uniform suction at the stationary disk. Archives of Mechanics. 1971; 23(1):27-36. <https://doi.org/10.1007/BF01593206>
5. Anjali Devi SP, Raghavachar MR. Magneto hydrodynamic stratified flow past a sphere. Int. J. Engng. 1982; 20(10):1169-77. [https://doi.org/10.1016/0020-7225\(82\)90097-0](https://doi.org/10.1016/0020-7225(82)90097-0)
6. Chandrashekhar DV, Rudraiah N. Electrically conducting fluid flow past an impermeable sphere embedded in a sparsely packed porous medium in the presence of transverse magnetic field. Proc. of 12th A. C. Fluid Mech. 2008; 1-4.
7. Blerlom RV. Magnetohydrodynamic flow of a viscous fluid past a sphere. J. Fluid. Mech. 2006; 8(3):438-41.
8. Jayalakshamma DV, Dinesh PA and Sankar M. Analytical study of creeping flow past a composite sphere: solid core with porous shell in presence of magnetic field. Mapana J. of Sci. 2011; 10(2):11-24. <https://doi.org/10.12723/mjs.19.2>
9. Shukla P, Das S. Effect of uniform magnetic field on the motion of porous sphere in spherical container. J. of Appl. Mathematics and Fluid Mechanics. 2015; 7(1): 51-6.
10. Ghosh S, Sarkar S, Sivakumar R, Sekhar TVS. Full magnetohydrodynamic flow past a circular cylinder considering the penetration of magnetic field. Physics of Fluids. 2018; 30(8):87-102. <https://doi.org/10.1063/1.5040949>
11. Pantokratoras A, Fang T. Flow of a weakly conducting fluid in a channel filled with a porous medium. Trans Porous med. 2010; 83:667-76. <https://doi.org/10.1007/s11242-009-9470-6>
12. Dhiman N, Awasthi M, Singh MP. Electro hydrodynamic Kelvin-Helmholtz instability of cylindrical interface through porous media. Int. J. Fluid Mechanics Research. 2013; 40(5):455-67. <https://doi.org/10.1615/InterJFluidMechRes.v40.i5.80>
13. Srivastava BG, Deo S. Effect of magnetic field on the viscous fluid flow in a channel filled with porous medium of variable permeability. Applied Mathematica and Computation. 2013; 219(17):8959-64. <https://doi.org/10.1016/j.amc.2013.03.065>
14. Yeh S, Tsai-Jung Chen, Leong JC. Analytical solution for MHD flow of a magnetic fluid within a thick porous annulus. J. Appl. Mathematics. 2014. <https://doi.org/10.1155/2014/931732>
15. Yadav S, Sharma PR. Effects of porous medium on MHD fluid flow along a stretching cylinder. Annals of Pure and Appl. Mathematics. 2014; 6(1):104-13.

16. Sharma MK, Singh K, Kumar A. MHD flow and heat transfer through a circular cylinder partially filled with non-Darcy porous media. *IJITEE*. 2014; 4(7).
17. Cauhan TS, Chauhan IS, Shikha. Flow of a viscous fluid through a porous circular pipe in the presence of magnetic field. *Mathematica Aeterna*. 2015; 5(2):395-402.
18. Verma VK, Gupta AK. Analytical solution of the flow in a composite cylindrical channel partially with a porous medium in the presence of magnetic field. *Special topics and reviews in porous media*. 2017; 8(1):39-48. <https://doi.org/10.1615/SpecialTopicsRevPorousMedia.v8.i1.30>
19. Ansari IA, Deo S. Magnetohydrodynamic viscous fluid flow past a porous sphere embedded in another porous medium. *Special topics and reviews in porous media*. 2018; 9(2):191-200. <https://doi.org/10.1615/SpecialTopicsRevPorousMedia.v9.i2.70>
20. Namdeo RP, Gupta BR. Slip at the surface of slightly deformed sphere in MHD flow. *Special Topics & Reviews in Porous Media: An International Journal*. 2022; 13(1):1-14. <https://doi.org/10.1615/SpecialTopicsRevPorousMedia.2021038694>