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The Combined Effect of Coriolis Force and Double Diffusive for the Convection in a Rectangular Isotropic and Anisotropic Porous Channel

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Abstract

The combined effects of Coriolis force, heat, and mass diffusion for the convection in a horizontal rectangular channel filled with fluid in both isotropic and anisotropic porous channels is studied in the three-dimensional case for which the governing equations with the boundary conditions are derived with the help of the following assumptions: the fluid is incompressible; the flow is axisymmetric about the y-axis; the fluid is steady; thermal diffusion is larger than the viscous diffusion. With the help of the stream function, the resultant equations are made dimensionless, and non-dimension parameters like R_{ac}, Rs along with anisotropic aspect ratio of permeabilities and thermal diffusivities are introduced. Later, the resulting linear partial differential equations are solved by the method of Fourier series analysis. The effect of the non-dimensional numbers plays a vital role in controlling parameters like velocity, temperature, and concentration for both isotropic and anisotropic cases. The expression for critical Rayleigh number has been established for both isotropic and anisotropic cases. Various computations are carried out in order to study the effects of velocity, temperature, concentration, and Critical Rayleigh number for different values of non-dimensional parameters. The obtained results are in good agreement with previous literary works and also have a wide application in the real world which can be seen in different industries.

Keywords: *Coriolis Force, Double Diffusion, Fourier Analysis*

1.0 Introduction

Natural convection is the term used to describe the motion of a fluid due to buoyance as a result of variations in density and temperature difference. Variation in density can also be caused not only by temperature differences but also due to concentration differences. Double diffusive convection is the fluid flow caused by buoyance because of both temperature and concentration gradient. This paper is a study of the flow in a rectangular enclosure and study the convection generated by buoyancy in conducting and non-conducting fluids. Long ago, oceanographers started studying the effects of double-diffusive convection. Linear stability gave rise to much of the research in this field where a simple salt-stratified fluid is heated from below. Buoyance-driven flows are studied in many natural and industrial processes and have peaked interest in recent times. Such flows are called convective flow where the temperature and concentration differences cause a variation of density.

There are three types of convection namely mixed convection, natural convection, and forced convection wherein the difference lies in the nature of the generation of the flow. The external applied force is a known aspect

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in the case of forced convection. Natural convection is a result of the interaction of density difference with either some force field or generally gravity and is linked with the dependence on temperature and concentration field. Mixed convection is a combination of forced and natural convection. Considering the mass and heat transfer process mathematically, we decouple the flow fields from the heat and mass transfer equation in forced convection it can be attained independent of the concentration and temperature fields. This means that the energy, momentum, and species equations are decoupled. This cannot be achieved in the case of natural convection and the temperature and concentration fields must be considered simultaneously where the energy, momentum, and species equations are coupled.

In our study, we find the Rayleigh number and the critical Rayleigh number for different conditions for both isotropic and anisotropic cases. The Liapunov method is widely used for the same when there are non-linear perturbations[1]. A study of superposed couple stress fluids in a porous medium was conducted by Sunil Sharma and Chandel[2]. The effect of heating and non-linear temperature distribution was studied by Neild[3]. Later, Speigal and Veronis deduced just by the Taylor number, Prandtl number, and Rayleigh number the convection in a rotating fluid layer. Hence the Rayleigh number will also be dependent on the Taylor number and the Prandtl number[4]. Chandrashekar studied the instability of a plane interface between two incompressible viscous fluids of different densities. Mohammadein and El-Shaer numerically analyzed how variable permeability and porosity effects convective flow from the vertical surface[5]. Temperature contours and Nusselt numbers also affect the flow of fluid during double-diffusive convection and were studied by Vijaya Kumara V. M[6]., Beg *et al*[7]., investigated the combined heat and mass transfer by the mixed magneto-convective flow of an electrically conducting flow along a moving radiating vertical flat plate with hydrodynamic slip and thermal convective boundary conditions.

2.0 Objective and Scope

Thanks to the significance of heat and mass transfer in the designing of reactors and also in energy storage, many researchers have started their study in this field and have discovered many applications such as

1. Furnaces, nuclear reactors, heat exchangers, salt gradient solar collectors.
2. Multi-layered walls, and double windows in building systems.
3. Material processing.
4. Astro and geophysical phenomena.
5. Natural convection processes in nature.

These are examples and mathematical models of modern-day science and technology. In most of the cases, buoyance is the main factor affecting the flow.

The following are the three types of mechanisms used to control convection

1. Usage of concentration gradient in two-component systems
2. Control of convection by slipping porous lining using a thin lining on the vertical wall
3. Usage of non-uniform applied temperature and concentration gradient control convection.

The first case is an example of double diffusion convection where the flow is by simultaneous concentration and temperature difference in the presence of gravity which causes changes in density. Oceanographers started the study of double diffuse convection owing to its vast applications. The theoretical work was developed from the theory of linear stability for salt-stratified fluid in infinite horizontal rectangular geometry which will be heated from below and salted from above. In a porous medium, no-slip boundary conditions cannot be applied because of the roughness of the porous material and the transfer of momentum from the fluid layer to the lining. Slip will exist only at the intersection of the porous lining and the fluid layer and is proportional to the tangential stress. Though temperature and concentration gradient were the core of the studies on single and double diffusivity, many of the problems include convection. For example, sudden heating or cooling of boundaries of geophysical applications is due to the presence of heat and concentration sources. Sutton[8] did research on convection in isotropic porous channels with perfectly insulation boundaries. Beck, Strauss Schuber, and Horne[9] studied convection in isotropic porous media which were confined in rectangular boxes.

3.0 Applications

There is a wide range of applications of the topic under study which include nuclear power plants, solar ponds,

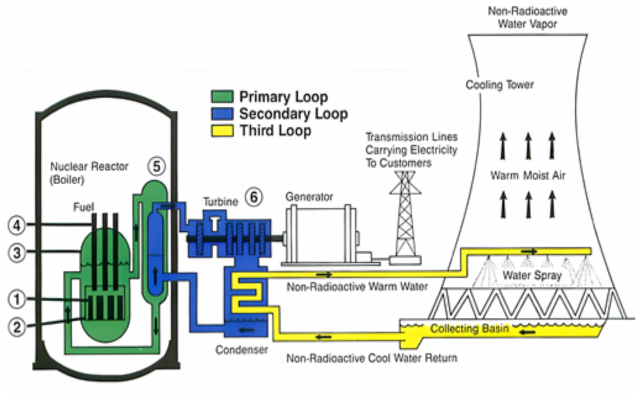

Figure 1. Nuclear power plant.

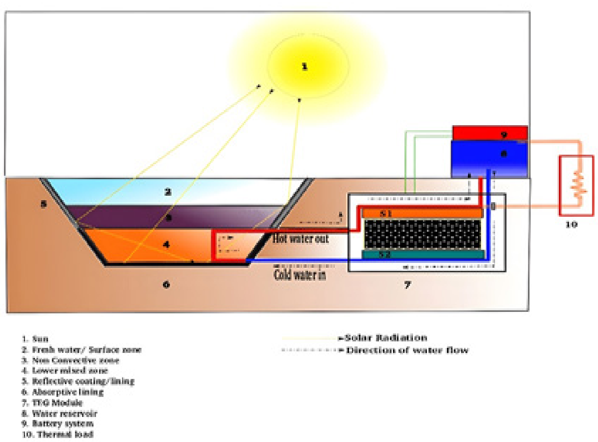

Figure 2. Nuclear power plant.

geophysical applications, etc. Two such applications that can be seen are in solar ponds and nuclear power plants.

The main principle behind the working of the solar pond is the thermal diffusivity and concentration gradient of the salt present in the pond. The bottommost layer is heated more thanks to the presence of salt and a decreasing temperature gradient can be observed from the bottom up. The heated stream at the bottom of the pond is then flown into another chamber where the steam produced is used to rotate a turbine. This energy produced can be used to generate electricity either by a thermo-electric device or a turbine.

The working principle in nuclear power plants is similar to that of the solar pond where instead of the sun, the heat source will be a radioactive power source). Nuclear reactors use a variety of fuels like coated particles, oxides, and carbides as well as radioactive metals like Plutonium and Thorium Uranium is most commonly used as it is more abundant than other metals[10]. Heat is produced by the process of nuclear fission which is used to heat the fluid (water). The steam generated turns the turbine motor and produces electricity. After cooling, the water the then collected in the cooling tower which will be reused for the process.

In the case of solar ponds, one can observe the changes in density gradient, temperature gradient, and concentration gradient as well. Likewise, in the case of nuclear power plants, the same is observed. The effect of the Coriolis force can be observed in the rotatory motion of the turbines in producing electricity. In this project, a miniature version of the flow and the phenomena happening is considered and worked on.

4.0 Mathematical Formulation

In this study, we consider 3-dimensional thermal convection in an incompressible fluid-saturated porous media which is confined in a horizontal rectangular channel rotating with an angular velocity of Ω in the vertical z direction. We assume that the walls are impermeable and conducting in order to establish a non-uniform temperature gradient in z direction. The channel under consideration is of height 'h and width 'a' measured in the Cartesian coordinate system with the x-axis as the horizontal and the z-axis as the vertical. The walls of the

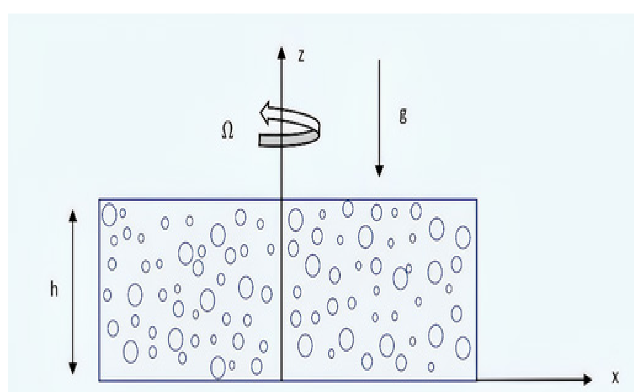

Figure 3. Mathematical formulation.

channel are z = 0 and z = h in the vertical direction and x=a/2 and x= -a/2.

The governing equations for the assumed problem are:

Conservation of Mass

$$\nabla.\vec{q} = 0 \tag{1}$$

Conservation of Energy

$$c\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa_x \frac{\partial^2 T}{\partial x^2} + \kappa_y \frac{\partial^2 T}{\partial y^2} + \kappa_z \frac{\partial^2 T}{\partial z^2} \tag{2}$$

Conservation of Momentum

$$\rho\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} + 2\Omega\hat{k} \times \vec{q}\right] = -\nabla p + \rho\hat{g} - \mu\hat{g} + \mu\nabla^2\vec{q} \tag{3}$$

Conservation of Mass Flux

$$\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = \sigma_x \frac{\partial^2 S}{\partial x^2} + \sigma_y \frac{\partial^2 S}{\partial y^2} + \sigma_z \frac{\partial^2 S}{\partial z^2} \tag{4}$$

Equation of State

$$\rho = \rho_0[1 - \beta(T - T_0) + \alpha(S - S_0)] \tag{5}$$

where the Darcy velocity is the volumetric flow rate of fluid per unit area, p is the pressure, ρ is the density, μ is viscosity, κ_x, κ_y, κ_z are thermal diffusivities. k_x, k_y, k_z are permeabilities, σ_y, σ_z are the mass flux diffusivities in x, y, z directions. T_0 is the reference temperature, T is the temperature at the point of fluid. S_0 is reference concentration. ω is the angular velocity of the system and $\vec{g}$ is the acceleration due to gravity.

The basic state of the system is given by

$$\vec{q} = (u, v, w) = (0,0,0),$$

$$\vec{g} = (0,0,-g) \text{ and } \vec{\omega} = (0,0,\omega)$$

A fluctuation is made which will be a function of x, y, z and t where the perturbed equations are given by $u = u'$; $v = v'$; $w = w'$; $\rho = (z) + \rho'$;
$p = \vec{p}\,(z) + p'$; $\vec{T}\,T = (z) + T$.

T_0 and $T_0 + \Delta T$ are the isothermal temperatures maintain at the upper and lower boundaries and T_0 and $T_0 + \Delta T$. The concentrations are maintained at S_0 and $S_0 + \Delta S$.

ΔT and ΔS are temperature and concentration differences which are both positive. This is under the assumption that tall the boundaries are perfectly heat conducting and impermeable.

The boundary conditions are $T = T_0 + \Delta T$, $S = S_0 + \Delta S$ on $z=0$ and $T = T_0$, $S = S_0$ on $z = h$.

The side walls are salted and uniformly heated. This means that the motion- less conductions state will exist if the concentration distribution and the static temperature are independent of x and linearly dependent on z. therefore the temperature and the concentration can be expressed as

$T = [T_0 + \Delta T\,(1 -)] + \theta$ and $S = [S_0 + \Delta S\,(1 - z/h)] + s$.

After removing the primes, the perturbed equations will be

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2\Omega v\right] = -\frac{\partial p}{\partial x} - \frac{\mu}{k_x}u + \mu\nabla^2 u$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + 2\Omega u\right] = -\frac{\partial p}{\partial y} - \frac{\mu}{k_y}v + \mu\nabla^2 v$$

$$\rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right] = -\frac{\partial p}{\partial z} - \frac{\mu}{k_z}w + \rho g + \mu\nabla^2 w$$

$$c\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \kappa_x \frac{\partial^2 T}{\partial x^2} + \kappa_y \frac{\partial^2 T}{\partial y^2} + \kappa_z \frac{\partial^2 T}{\partial z^2}$$

$$\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial x} + v\frac{\partial S}{\partial y} + w\frac{\partial S}{\partial z} = \sigma_x \frac{\partial^2 S}{\partial x^2} + \sigma_y \frac{\partial^2 S}{\partial y^2} + \sigma_z \frac{\partial^2 S}{\partial z^2}$$

$$\rho = \rho_0[1 - \beta(T - T_0) + \alpha(S - S_0)] \tag{6}$$

Density is neglected in all terms of momentum equation except in the buoyancy term which gives rise to the Boussinesq approximation wherein the product of the variation of density and velocity is very small. And since the flow is also axi-symmetric about the y-axis, all the partial derivatives with respect to y will be zero. Assuming the Prandtl-Darcy number to be large, inertia and the viscous term in the momentum equation is neglected. Hence, we get

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial x}+\frac{\nu}{k_x}v-2\Omega v=0$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial y}+\frac{\nu}{k_y}v+2\Omega u=0$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial z}+\frac{\nu}{k_z}w-\frac{\rho}{\rho_0}g=0$$

$$c\frac{\partial T}{\partial t}+v.\nabla T=\kappa_x\frac{\partial^2 T}{\partial x^2}+\kappa_z\frac{\partial^2 T}{\partial z^2}$$

$$\frac{\partial S}{\partial t}+v.\nabla S=\sigma_x\frac{\partial^2 S}{\partial x^2}+\sigma_z\frac{\partial^2 S}{\partial z^2}$$

$$\rho=\rho_0[1-\beta(T-T_0)+(S-S_0)] \quad (7)$$

We introduce the two-dimensional stream functions which are functions of x and z alone as the flow is axi-symmetric about the y-axis satisfying the continuity equation. The functions will be of the form ψ=(x,z) and u=∂ψ/∂z; w=-∂ψ/∂x . The equations will then be non dimensionalised using the following transformations.

$$u=\frac{\kappa_z a u^*}{h^2};\ v=\frac{\kappa_z a v^*}{h^2};\ w=\frac{\kappa_z w^*}{h};\ t=\frac{ch^2t^*}{\kappa_z};$$

$$x=ax^*;\ y=ay^*;z=hz^*;$$

$$\psi=\frac{\kappa_z a\psi^*}{h};\ \theta=\Delta T\theta^*;\ T_0=\Delta TT_0^*;$$

$$p=\frac{\nu k_z\rho_0 p^*}{k_x};\ S=\Delta SS^*$$

After substituting the above transformations and by eliminating density, pressure and '*', we get the governing equations as

$$\left(\xi\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\psi+\xi R_a\frac{\partial\theta}{\partial x}-\xi R_s\frac{\partial s}{\partial x}-T_aD_a\frac{\partial v}{\partial z}=0$$

$$\chi\frac{\partial v}{\partial z}+T_aDa\frac{\partial^2\psi}{\partial z^2}=0$$

$$P_c\left(\zeta\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)s-\frac{\partial\psi}{\partial t}=\frac{\partial s}{\partial t}\frac{1}{c}+\frac{\partial\psi}{\partial x}\frac{\partial s}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial s}{\partial z}$$

$$\left(\eta\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\theta-\frac{\partial\psi}{\partial x}=\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial z} \quad (8)$$

R_a is the Rayleigh number and R_s is the Darcy Solutal Rayleigh number and defined by

$$R_a=\frac{\beta g\Delta Tk_zh}{k_z\nu}\ and\ R_s=\frac{\alpha g\Delta Sk_zh}{k_z\nu}$$

$$\xi=\frac{k_x}{k_z}\left(\frac{h}{a}\right)^2,\eta=\frac{\kappa_x}{\kappa_z}\left(\frac{h}{a}\right)^2, \quad (9)$$

And ξ, ζ, η and χ are the anisotropy aspect ratios of permeabilities and diffusivities of temperature given by

$$\zeta=\frac{\sigma_x}{\sigma_z}\left(\frac{h}{a}\right)^2,\qquad \chi=\frac{k_x}{k_z} \quad (10)$$

The boundary conditions for a perfectly heat conducting impermeable boundary leads to

$$\psi=\theta=\frac{\partial v}{\partial z}=0$$

$$on\ \begin{cases} x=-\frac{1}{2}, & x=\frac{1}{2} \quad 0<z>1 \\ z=0, & z=1 \quad -\frac{1}{2}<x<\frac{1}{2}\end{cases} \quad (11)$$

5.0 Linear Stability and Flow Patterns

The onset of convection is described by the linear versions of the governing equations

$$\psi=e^{\sigma T}\left[\frac{1}{2}C_0+\sum_{n=1}^{\infty}C_n(x)cosn\pi z+D_n(x)sinn\pi z\right]$$

$$\theta=e^{\sigma T}\left[\frac{1}{2}F_0+\sum_{n=1}^{\infty}F_n(x)cosn\pi z+G_n(x)sinn\pi z\right]$$

$$v=e^{\sigma T}\left[\frac{1}{2}A_0+\sum_{n=1}^{\infty}A_n(x)cosn\pi z+B_n(x)sinn\pi z\right]$$

$$S=e^{\sigma T}\left[\frac{1}{2}S_0+\sum_{n=1}^{\infty}S_n(x)cosn\pi z+H_n(x)sinn\pi z\right] \quad (12)$$

Here *Cn, Dn, Fn, Gn, An, Bn, Sn* and *Hn* are functions of x alone and is the growth rate. The boundary conditions will be satisfied if *Cn = Fn = Hn = Bn = 0* for all values of x. Comparing the sin(nπz) terms for and and cos(nπz) for v and S and converting into linearised governing equations, we get the following ordinary differential equations

$$\left(\xi \frac{\partial^2}{\partial x^2} - n^2 \pi^2\right) D_n + \xi R_a \frac{dG_n}{dx} - \xi R_s \frac{dH_n}{dx}$$
$$+T_a Da\, n\pi A_n = 0$$
$$\chi A_n + T_a Da\, n\pi D_n = 0$$
$$P_c\left(\zeta \frac{d^2}{dx^2} - n^2\pi^2\right) H_n - \frac{dD_n}{dx} = \sigma H_n$$
$$\left(\eta \frac{d^2}{dx^2} - n^2\pi^2\right) G_n - \frac{dD_n}{dx} = \sigma G_n$$

(13)

The boundary conditions for *Dn, Gn, An* and *Hn* are given by

$$D_n\left(\frac{1}{2}\right) = D_n\left(-\frac{1}{2}\right) = 0;\ A_n\left(\frac{1}{2}\right) = A_n\left(-\frac{1}{2}\right) = 0;$$
$$G_n\left(\frac{1}{2}\right) = G_n\left(-\frac{1}{2}\right) = 0;\ H_n\left(\frac{1}{2}\right) = H_n\left(-\frac{1}{2}\right) = 0$$

(14)

Operators on the LHS of the boundary conditions are self-adjoint and we can come to the conclusion that σ is real. R_{ac} is a function of ξ, ζ, η and χ and upon using the condition σ=0, we get the Critical Rayleigh number. We then get

$$\psi = D_n \sin n\pi z$$
$$\theta = G_n \sin n\pi z$$
$$v = A_n \cos n\, \pi z$$
$$S = H_n \sin n\, \pi z$$

(15)

The system of equations are subjected to the boundary conditions and will represent the self-adjoint Eigen value problem with Eigen value with R_{ac} and R_a being the smallest. The problem under consideration can be divided into two cases

Isotropic case where ξ=ζ=η=χ

Anisotropic case where ξ≠ζ≠η≠χ

Case 1: Isotropic Case

The condition ξ=ζ=η=χ can only be satisfied when the ratio of the horizontal and vertical components of the permeability and the thermal diffusivity are equal.

i.e $$\frac{k_x}{k_z} = \frac{\kappa_x}{\kappa_z} = \frac{k_x}{k_y}$$

In isotropic case, we get the general solution of D_n and G_n as

$$D_n(x, R_a) = C_1 \cos px + C_2 \sin px + C_3 \cos qx$$
$$+C_4 \sin qx$$
$$G_n(x, R_a) = s\begin{bmatrix} rC_1 \sin p\,x - rC_2 \cos p\,x + \\ C_3 \sin q\,x - C_4 \cos q\,x \end{bmatrix}$$

(16)

Where p and q are

$$p = \frac{1}{2\sqrt{\xi}}\sqrt{\begin{matrix}\xi\left(R_a - \frac{R_s}{P_c}\right) - (n\pi)^2\left[\xi + \eta\left(1 + \frac{T_a^2 Da^2}{\chi}\right)\right] \\ -2(n\pi)^2\sqrt{\xi\eta\left(1 + \frac{T_a^2 Da^2}{\chi}\right)}\end{matrix}}$$
$$-\sqrt{\begin{matrix}\xi\left(R_a - \frac{R_s}{P_c}\right) - (n\pi)^2\left[\xi + \eta\left(1 + \frac{T_a^2 Da^2}{\chi}\right)\right] \\ -2(n\pi)^2\sqrt{\xi\eta\left(1 + \frac{T_a^2 Da^2}{\chi}\right)}\end{matrix}}$$

(17}

$$q = \frac{1}{2\sqrt{\xi}}\sqrt{\begin{matrix}\xi\left(R_a - \frac{R_s}{P_c}\right) - (n\pi)^2\left[\xi + \eta\left(1 + \frac{T_a^2 Da^2}{\chi}\right)\right] \\ -2(n\pi)^2\sqrt{\xi\eta\left(1 + \frac{T_a^2 Da^2}{\chi}\right)}\end{matrix}}$$
$$-\sqrt{\begin{matrix}\xi\left(R_a - \frac{R_s}{P_c}\right) - (n\pi)^2\left[\xi + \eta\left(1 + \frac{T_a^2 Da^2}{\chi}\right)\right] \\ -2(n\pi)^2\sqrt{\xi\eta\left(1 + \frac{T_a^2 Da^2}{\chi}\right)}\end{matrix}}$$

(18)

When $C_2 = C_4 = 0$, we get

$$(1 - r)\sin\left(\frac{p + q}{2}\right) - (1 + r)\sin\left(\frac{p - q}{2}\right) = 0$$

(19)

And when $C_1 = C_3 = 0$, we get

$$(1-r)\sin\left(\frac{p+q}{2}\right) + (1+r)\sin\left(\frac{p-q}{2}\right) = 0 \tag{20}$$

Substituting the above equations, we get

p-q=2mπ (21)

And

$$R_a = 4\pi^2\left[\xi m + \frac{n^2}{2}\left(\frac{T_a^2 Da^2}{2\xi} + \sqrt{1+\frac{T_a^2 Da^2}{\xi}}\right)\right] + \frac{R_s}{P_c} \tag{22}$$

Where n = 1, 2, 3… and m = 1, 2, 3…

The smallest value of equation (22) gives the critical Rayleigh number when n = 1 and m = 1

$$R_{a_c} = 4\pi^2\left[\xi + \frac{1}{2}\left(\frac{T_a^2 Da^2}{2\xi} + \sqrt{1+\frac{T_a^2 Da^2}{\xi}}\right)\right] + \frac{R_s}{P_c} \tag{23}$$

In the limiting case $h/a \to 0$ and $T_a \to 0$ the channel will become an infinite horizontal porous layer. We get the Critical Rayleigh number R_{ac} as $R_a = 4\pi^2$ which is slightly different from equation (22) for a channel. Anisotropic case where $\xi \neq \zeta \neq \eta \neq \chi$ Therefore, the two sets of solutions are

$$\psi^{\{(1)\}} = Q \sin\left(\pi\sqrt{\left\{1+\frac{1}{\xi}\sqrt{\left\{1+\frac{T_a^2 Da^2}{\xi}\right\}}\right\}}x\right)\cos\pi x \sin\pi z$$

$$\theta^{(1)} = -Q\, s\cos\left(\sqrt{1+1/\xi\sqrt{\left(1+\frac{(T_a^2 Da^2)}{\xi}\right)}}x\right) cos\pi x \sin n\pi z$$

$$v^{(1)} = -\frac{T_a D_a \pi}{\xi} Q \sin\left(\pi\sqrt{\left\{1+\frac{1}{\xi}\sqrt{\left\{1+\frac{T_a^2 Da^2}{\xi}\right\}}\right\}}x\right)\cos\pi x \sin\pi z$$

$$s^{(1)} = -Q\, t\cos\left(\sqrt{1+\frac{1}{\xi}\sqrt{\left(1+\frac{(T_a^2 Da^2)}{\xi}\right)}}x\right) cos\pi x \sin n\pi z \tag{24}$$

And

$$\psi^{(2)} = \frac{s}{a}\cos\left(\pi\sqrt{\left\{1+\frac{1}{\xi}\sqrt{\left\{1+\frac{T_a^2 Da^2}{\xi}\right\}}\right\}}x\right)\cos\pi x \sin\pi z$$

$$\theta^{(2)} = -S\sin\left(\sqrt{1+\frac{1}{\xi}\sqrt{\left(1+\frac{(T_a^2 Da^2)}{\xi}\right)}}x\right) cos\pi x \sin n\pi z$$

$$v^{(2)} = -\frac{T_a D_a S\pi}{\xi s} Q \cos\left(\pi\sqrt{\left\{1+\frac{1}{\xi}\sqrt{\left\{1+\frac{T_a^2 Da^2}{\xi}\right\}}\right\}}x\right)\cos\pi x \sin\pi z$$

$$s^{(2)} = \frac{St}{s}\cos\left(\sqrt{1+\frac{1}{\xi}\sqrt{\left(1+\frac{(T_a^2 Da^2)}{\xi}\right)}}x\right) cos\pi x \sin n\pi z \tag{25}$$

Case 2: Anisotropic Case

In anisotropic case , and we get the general solution for D_n and G_n as

$$D_n(x, R_a) = C_1 \cos px + C_2 \sin px + C_3 \cos qx + C_4 \sin qx$$

$$G_n(x, R_a) = s\begin{bmatrix} rC_1 \sin p\,x - rC_2 \cos p\,x \\ +C_3 \sin q\,x - C_4 \cos q\,x \end{bmatrix} \tag{26}$$

Calculations are carried out similar to the isotropic case and we obtain the R_{ac} as

$$R_{a_c} = \pi^2\left[\left(1+\sqrt{\frac{\eta}{\xi}\left(1+\frac{T_a^2 Da^2}{\chi}\right)}\right)^2 + 4\eta\right] + \frac{R_s}{P_c} \tag{27}$$

The behaviour for different Taylor numbers and anisotropic rations of permeability and thermal diffusivity are shown in the results and discussions section. The criteria for onset of convection is given by the above equation. The onset of convection is characterised by the supercritical Rayleigh number at $R_a = R_c$. The two sets of solutions are

$$\psi^{(1)} = 2C_2 sinkx \cos\pi x \sin\pi z$$

$$\theta^1 = -C_2\, s\{(r+1)\cos kx\ \cos\pi x + (1-r)\sin\pi kx \sin\pi x\}\sin\pi z$$

$$S^{(1)} = -C_2 t_1\begin{Bmatrix}(r+1)coskx\ cos\pi x + \\ (1-r)\ sin\pi kx\ sin\pi x\end{Bmatrix} sin\pi z$$

$$v^1 = -\frac{2T_a D_a \pi C_2}{\chi} sinkx \cos\pi x \sin\pi z \tag{27}$$

And

$$\psi^{(2)} = 2C_1 sinkx \cos\pi x \sin\pi z$$

$$\theta^2 = -C_1 s\{(r+1)\cos kx \ \cos\pi x + (1-r)\sin\pi kx \sin\pi x\}\sin\pi z$$

$$S^{(2)} = -C_1 t_1 \begin{Bmatrix} (r+1)coskx \ cos\pi x + \\ (1-r)\ sin\pi kx \ sin\pi x \end{Bmatrix} sin\pi z$$

$$v^2 = -\frac{2T_a D_a \pi C_1}{\chi} sinkx \cos\pi x \sin\pi z \tag{28}$$

6.0 Graphs and Isotherms

Isotropic case

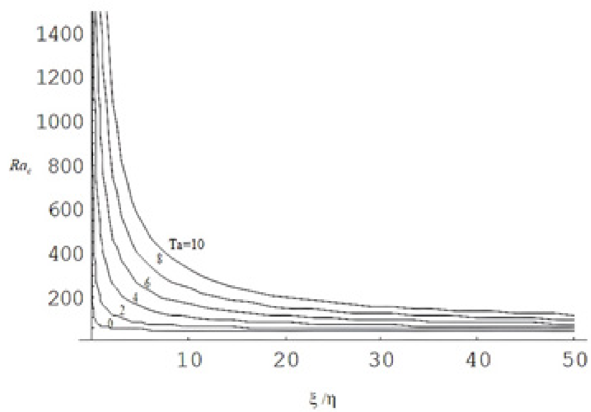

Figure 4. Graph (Isotropic case).

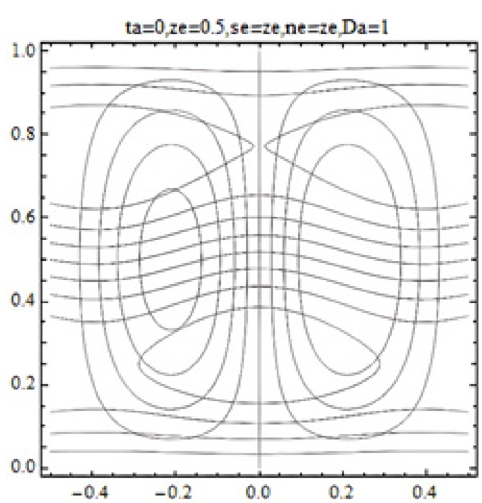

Figure 5. Isotherm 1 (Isotropic case).

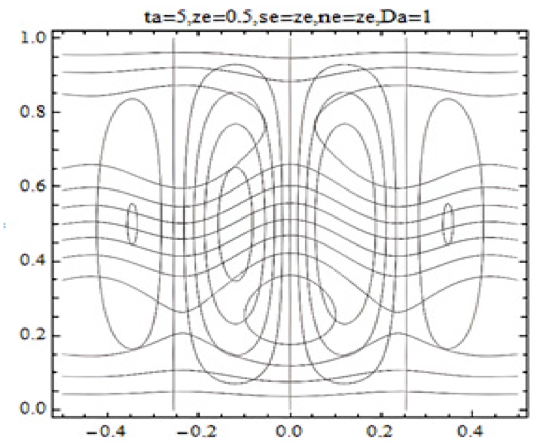

Figure 6. Isotherm 2 (isotropic case).

Anisotropic case

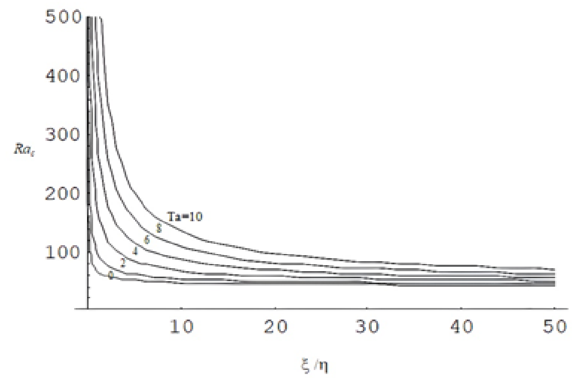

Figure 7. Graph (anisotropic case).

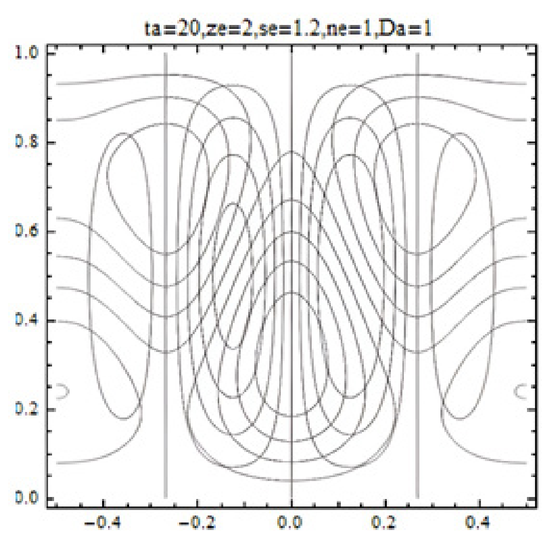

Figure 8. Isotherm 1 (Anisotropic case).

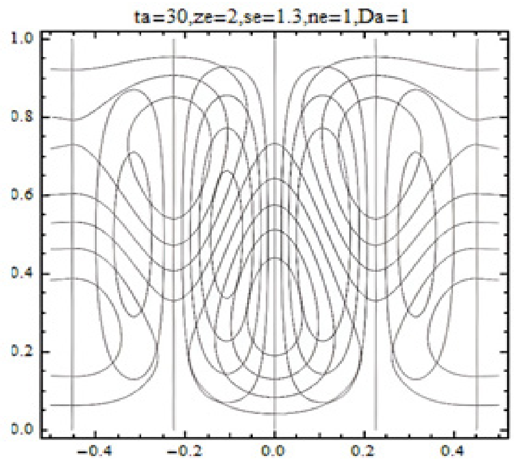

Figure 9. Isotherm 2 (Anisotropic case).

7.0 Observations

- The number of cells in both cases were found to be increasing as the Taylor number increased.
- The isothermal lines represent the increase in the oscillatory flow behavior with rotation with anisotropy.
- The number of cells decrease with increase in the aspect ratio and the thermal diffusivity in anisotropic case.
- The critical Rayleigh number is inversely proportional to the ratio of
- It also increases with the increase in the Taylor number, solutal Rayleigh number and with the effect of rotation.

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