

Investigation of Temperature Distribution and Heat Transfer Rate of Moving Fin with Radiation and Magnetic Field Effects

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Abstract

Fins, comprising metal strips strategically affixed to surfaces, play a pivotal role in augmenting heat transfer processes in various engineering applications. Fins are commonly used in plethora of heat transfer applications and also utilized for the prevention and control of thermal damage in many mechanical and electronic devices. This study focuses on thermal responses of moving fin with the influence of magnetic field and radiation. The developed thermal model is solved analytically and parametric investigation is carried out. Influence of radiation parameter, geometry parameter, Hartmann number and Peclet number on thermal variation is interpreted through graphical representations and results are concluded.

Keywords: Magnetic Field, Moving Fin, Radiation

1.0 Introduction

Systems used in engineering and several aspects of life are all affected by heat transfer. Heat transfer is mainly considered in the design of appliances which includes boilers, solar collectors, condensers, furnaces, heat exchangers, radiators and heaters. Even so, there are impacts of combined conduction and convection in a variety of aspects, the most important is the one that uses an extended surface, especially to increase or decrease heat transmission. Such emerging surfaces are frequently referred to as fins. By using the fins, surfaces that are cooled by gases can transfer heat more quickly, under forced or natural convection. To prepare finned surfaces, the thin metal sheet is wrapped, welded, or extruded on an area. The principal objective of the designer during fin production is to enhance the size and cost of the fin. Frequently in the engineering

fields, it is preferable to have fins with thinner size is considered to analyze the heat transfer rates. Metals having a high thermal conductivity are employed in these circumstances. Metals, known for their excellent thermal conductivity properties, are frequently chosen as the primary material for fins in heat transfer applications. Metals such as aluminum, copper, and steel are favored due to their ability to quickly conduct heat away from the heat source and efficiently distribute it across the fin surface. This characteristic is crucial for maintaining optimal operating temperatures in heat-sensitive systems. Consequently, the investigation of heat transfer rate and temperature profile using a fin which is straight and moving with rectangular cross-section is the main objective of the current study.

Extensive researches have been conducted to analyze heat transfer using fins. Madhura *et al.*^{1,2} have discussed

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about the thermal analyzation of straight porous fin by considering variable thermal conductivity under influence of radiation and magnetic field. Oguntala *et al.*³ have utilized the effective method of iteration for the study of the convective- radiative porous fin structure with internal heat generation under the effect of uniform magnetic field. Asadi *et al.*⁴ have investigated the temperature distribution along fin with constant cross-sectional area. The researchers like Hossain *et al.*⁵, Mokheimer⁶ and Rao *et al.*⁷ have contributed their investigations by considering different aspects of fin.

The significance of the moving fin, the impact of radiation and magnetic fields are shown by the analysis of the above-mentioned investigations. Therefore, an attempt has been made in the present work to explore the temperature performance of a conductive-radiative straight moving fin with a rectangular cross-section geometry under the impact of magnetism and radiation. The analytical solutions for distribution of temperature are exhibited analytically and the non-dimensional parameters pertaining to temperature distribution have been analyzed through graphical representation.

2.0 Mathematical Formulation

The consideration of a conductive-radiative straight moving fin with a rectangular cross-section structure having width as w , length as b and thickness as t is exhibited in the Figure 1. The temperature is taken as T_b at fin base and environment temperature as T_∞ are

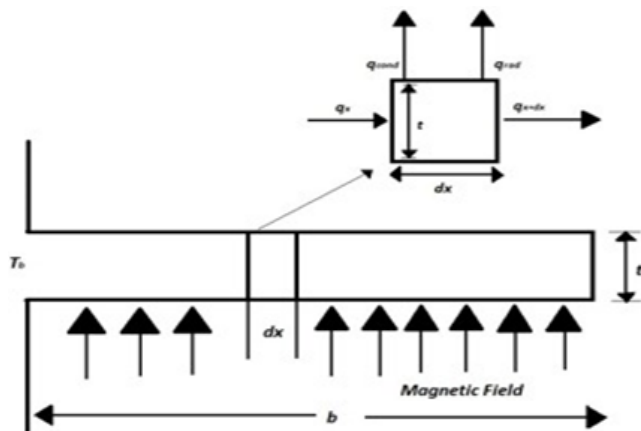


Figure 1. The geometry of straight fin with rectangular cross-section.

maintained. The magnetic field is applied uniformly in the direction of y and the electric field is assumed to be absent.

From the shown assumptions, the governing equation is given by

$$q_x - q_{x+dx} = hp(T - T_\infty) + \frac{J_c \times J_c}{\sigma} + u\rho C_p A_c \frac{dT}{dx} \quad (1)$$

where, $J_c = \sigma(E + V \times B)$ is conduction current density.

Vector form of energy flux along with conduction and radiation is obtained at the fin base is as given below,

$$Q_{\text{base}} = Q_{\text{cond}} + Q_{\text{rad}}. \quad (2)$$

Q_{cond} is considered using Fourier's law and is given as follows,

$$Q_{\text{cond}} = -A_c K(t) \frac{dT}{dx}. \quad (3)$$

Rosseland approximation is considered for Q_{rad} and is given by

$$Q_{\text{rad}} = -\frac{4\sigma_{st} A_c}{3\beta_r} \frac{dT^4}{dx}. \quad (4)$$

Using equations (3) and (4) in equation (1) we obtain,

$$\left[KA_c + \frac{16\sigma A_c T_\infty^3}{3\beta_r} \right] \frac{d^2 T}{dx^2} = hp(T - T_\infty) + \frac{J_c \times J_c}{\sigma} (T - T_\infty) + u\rho C_p A_c \frac{dT}{dx}. \quad (5)$$

$$\text{But, } \frac{J_c \times J_c}{\sigma} = \sigma B_0^2 u^2 \quad (6)$$

It is assumed that the variation of temperature in the flow is negligible in some conditions, then the T^4 can be expressed as,

$$T^4 \approx T_\infty^3 (4T - 3T_\infty). \quad (7)$$

$$\left[KA_c + \frac{16\sigma A_c T_\infty^3}{3\beta_r} \right] \frac{d^2 T}{dx^2} = hp(T - T_\infty) + \sigma u^2 B_0^2 (T - T_\infty) + u\rho C_p A_c \frac{dT}{dx}. \quad (8)$$

It is important to note that the fins tip is insulated and does not conduct heat; thus leads to the following boundary conditions,

$$T = T_b \text{ at } X = 0$$

$$\frac{dT}{dX} = 0 \text{ at } X = L$$

The dimensionless forms for physical quantities are considered as below,

$$\Theta = \frac{T - T_{\infty}}{T_b - T_{\infty}}, X = \frac{x}{b}$$

Using the above non-dimensional quantities, (8) reduces to

$$\frac{d^2\Theta}{dX^2} - \frac{Pe}{1 + 4R} \frac{d\Theta}{dX} - \frac{M^2 + H}{1 + 4R} \Theta = 0. \quad (9)$$

where,

$$R = \frac{4\sigma_{st}T_{\infty}^3}{3\beta_r K}, M^2 = \frac{hpb^2}{KA_c}, H = \frac{\sigma u^2 B_0^2 b^2}{KA_c}, Pe = \frac{u\rho c_p b}{\kappa}$$

$$R = \frac{4\sigma_{st}T_{\infty}^3}{3\beta_r K}, M^2 = \frac{hpb^2}{KA_c}, H = \frac{\sigma u^2 B_0^2 b^2}{KA_c},$$

The non-dimensionalized boundary conditions are provided by,

$$\Theta = 1 \text{ at } X = 0$$

$$\frac{d\Theta}{dX} = 0 \text{ at } X = 1 \quad (10)$$

Let us consider

$$A = \frac{Pe}{1 + 4R} \text{ and } B = \frac{M^2 + H}{1 + 4R} \quad (11)$$

The solution for (9) is

$$\frac{d^2\Theta}{dX^2} - A \frac{d\Theta}{dX} - B\Theta = 0 \quad (12)$$

By employing the boundary conditions, the above equation (12) reduces to

$$\Theta(X) = \frac{-m_2 e^{m_2} e^{m_1 X} + m_1 e^{m_1} e^{m_2 X}}{m_1 e^{m_1} - m_2 e^{m_2}} \quad (13)$$

3.0 Heat Transfer Rate

The equation for finding the rate of heat transfer from the base of the fin is given as:

$$q = -KA_c \left(\frac{d\Theta}{dX} \right)_{X=0}$$

$$q = -KA_c \left[\frac{(-m_1 m_2 e^{m_2 m_1 X}) + m_1 m_2 e^{m_1 m_2 X}}{m_1 e^{m_1} - m_2 e^{m_2}} \right]_{X=0}$$

$$q = \frac{-KA_c [m_1 m_2 (e^{m_2} - e^{m_1})]}{m_1 e^{m_1} - m_2 e^{m_2}}$$

4.0 Results and Discussion

The study of moving fin influenced by the effect of magnetic field with conduction and also radiation is executed. The mathematical model is non dimensionalized and the corresponding linear homogeneous differential equation of second order is resolved using suitable boundary conditions. Graphs are plotted to analyze the characteristics of temperature distribution. Variation of temperature with different non dimensionalize physical parameters like radiation parameter, as geometry parameter, Hartmann number and Peclet number are investigated. Further, variation of temperature

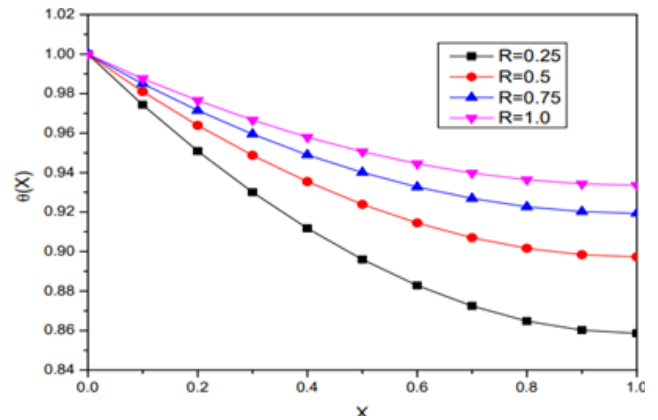


Figure 2. Variation of temperature distribution v/s radiation parameter.

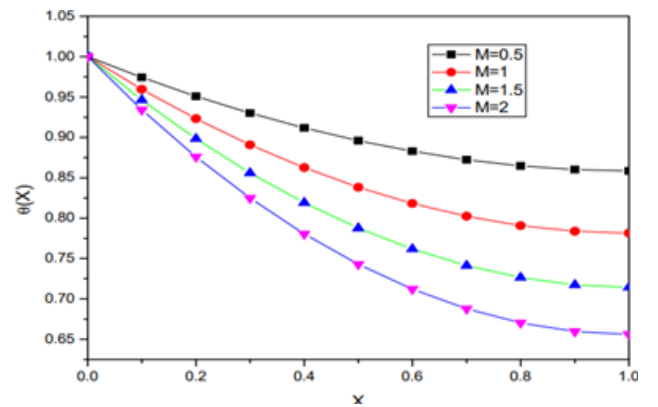


Figure 3. Variation of temperature distribution v/s Geometry parameter.

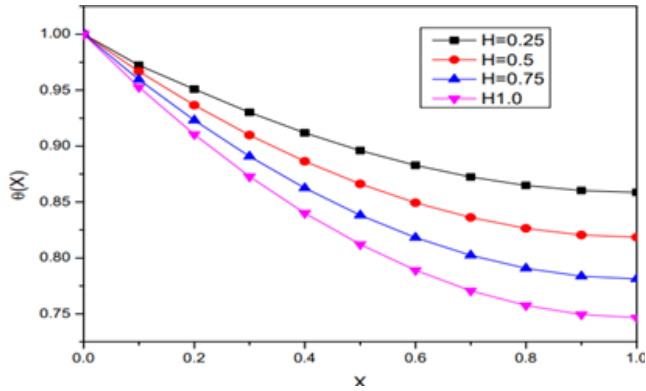


Figure 4. Variation of temperature distribution v/s Hartmann parameter.

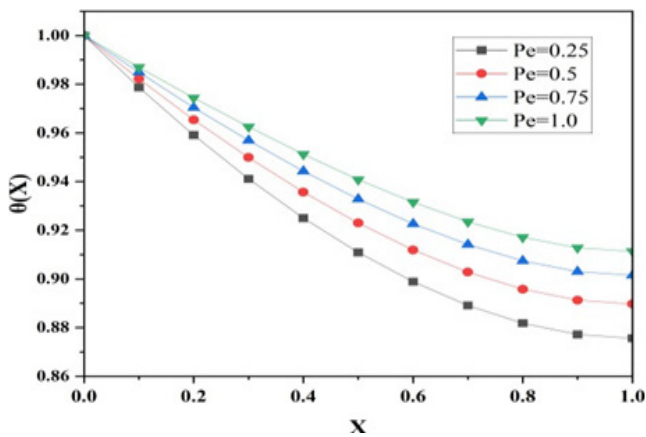


Figure 5. Variation of temperature distribution v/s Peclet parameter.

distribution for different materials like silver, copper and aluminum is carried out.

Figure 2 depicts the temperature distribution for different values of R, i.e., Radiation parameter. It is clearly examined that for higher values of R, temperature increases. Thus, R plays an vital role in the process of cooling the system.

Figure 3 exhibits the temperature curves for various range of Geometry parameter (M) values. It is clearly noted that there is a monotonically decreasing trend of temperature along the fin as M values rises. The reason for this behavior is interpreted as follows.

More the values of M intensify the heat transfer rate of the fin, as it is noted that for larger transfer of heat through convection in the fin leads to larger transfer of heat through fin by conduction mode also. This leads to temperature distribution enhancement in the fin and consequently, heat transfer rate.

Figure 4 describes the change of temperature for various values of H, i.e., Hartmann number. It is observed that temperature is inversely co-relate with Hartmann number. This trend is interpreted as, increasing the values of H leads to strengthening of the magnetic field impact on the system which results in decreasing the convective heat transfer process and in turn the average rate of heat transfer elevates in the fin. Moreover, presence of the magnetic field leads to increase in the strength of buoyancy force and fin produces more heat through convection. Accordingly, the heat transfer from the fin magnifies.

Figure 5 depicts the effects of Peclet number on velocity distribution. Peclet number generally represents the non-dimensional speed and it is defined as the ratio

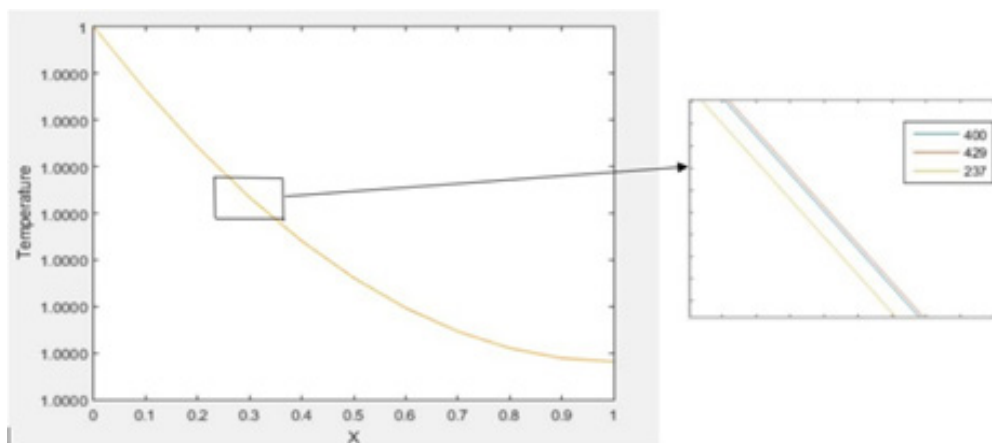


Figure 6. Variation of temperature distribution v/s thermal conductivity.

of advective heat and diffusive heat. As values of Peclet number increases, speed of the fin accelerates, which leads to, decelerates interaction effect between fin and the surrounding. Thus, Peclet number increases the thermal response of the fin.

Figure 6 represents the temperature distribution for different materials of the fin. In this study three different materials of the fins are considered, i.e., copper, silver and aluminum. Graphical representation elucidated that fin made up of copper material exhibits more temperature, then is silver fin in which aluminum material is used. This is due to the reason that copper exhibits more thermal conductivity than other materials.

5.0 Conclusion and Significance of the Study

This novel study describes an investigation of the temperature response of the straight moving fin with rectangular cross-section. By joining thin metal strips called as fins, the heat transmission between a hot surface and the surrounding fluid can be enhanced. These add-ons are attached to the hot surface to speed up the heat transfer. These results show that the moving fin profile plays a vital role in temperature distribution, especially near the fin tip. And as the fin moves faster, it is observed that there is cooling in the equipment. The results of different parameters for straight moving fin with rectangular cross-section on temperature distribution are analyzed by graphical representation.

- The radiation parameter (R) is directly proportional to the temperature.
- The temperature decreases as the geometric parameter (M) is increased.
- Impact of Hartmann number (H) is inversely proportional to temperature distribution of the fin.
- As the Peclet number (Pe) increases the temperature profile also increases.
- Higher values of thermal conductivity of different materials results in an increase in the temperature.

6.0 References

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Greek Symbols

Θ = Dimensionless temperature

σ = Electrical conductivity, $\Omega^{-1} \cdot m^{-1}$

σ_{st} = Stefan - Boltzmann constant, $W/m^2 \cdot K^4$

β_r = Rosseland mean absorption coefficient

ρ = Density of fin material, kg/m^3

Nomenclature

- A_c = Fin cross sectional area, m^2
 b = Fin length, m
 B_o = Magnetic field intensity, T
 c_p = specific heat, $m^2/s^2 \cdot K$
 M = Fin geometry parametry
 H = Hartmann number
 p = Fin perimeter, m
 J_c = Conduction current intensity, A
 T = fin temperature, K
 R = Radiation parameter
 h = coefficient of convective heat transfer, $J/m^2 \cdot K$
 T_b = Fin base temperature, K
 T_∞ = Surrounding medium temperature
 X = Dimensionless fin length